The determination of rock mass modulus variation with depth for weathered or jointed rock

R. K. Rowe

Faculty of Engineering Science, University of Western Ontario, London, Ont., Canada N6A 5B9

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Weathering or the variation in frequency and tightness of joints may result in an increase in mass modulus with depth for some rocks. This increase in modulus will continue until a depth is reached at which the rock behaves as a sound intact unit and the modulus will be relatively constant with depth below this point. In this paper, elastic solutions are presented for the deformation of such a rock mass due to a uniform or approximately rigid circular loading.

Two procedures are described for determining the rock mass modulus profile from plate load test results. The first procedure uses the results from three plate tests to infer the variation in modulus with depth. The second procedure uses the measured variation in displacement with depth below a single plate to infer the mass modulus variation. The application of the two procedures is illustrated by a worked example and by consideration of a field case where the inferred modulus is shown to be in good agreement with alternative modulus variation data.

Introduction

In the analysis of many geotechnical problems, it is convenient to idealize rock as being an elastic material. The suitability of this idealisation will depend upon the type and condition of the rock as well as the proposed loading or unloading conditions. Frequently, the rock will be jointed, weathered, and/or the response of the rock will be dependent upon the in situ stresses. Under these circumstances, an appropriate rock mass modulus, for use in analysis, is best determined from in situ testing.

The plate load test has been extensively used for estimating rock mass modulus. In some cases, a solid circular plate has been used and the deflections of the plate and the surrounding rock surface were monitored. Clearly, the relative rigidity of this plate will depend upon the rock mass modulus. Also, the interpretation of the measured displacement assumes that a suitable reference point on the rock surface can be determined (e.g., Van Heerden 1976). An alternative approach commonly adopted is to use a circular flat jack with a small hole in the centre. This arrangement provides a relatively uniform distribution of surface load and permits the determination of vertical displacement with depth beneath the loaded surface. In this case, the modulus may be determined from the relative displacement below the loaded plate.

The results of plate load tests have generally been interpreted using elastic solutions for the displacement of a rigid or flexible circular footing resting on a homogeneous elastic half-space (see Poulos and Davis 1974). This is clearly appropriate for a truly homogeneous rock mass; however, in many instances weathering or the frequency and tightness of joints will vary with depth leading to an increase in mass modulus until a depth is reached where the rock behaves as a sound intact rock. This may occur due to natural processes, or alternatively, excavation and construction may loosen rock near the surface reducing the modulus of the upper rock compared with that of the rock below. In either case, the rock profile may be idealized as having a linear increase in modulus with depth between the surface and some depth, z, and a relatively constant modulus below this as shown in Fig. 1.

Theoretical solutions for the surface settlement of a footing on a material where the modulus increases...
linearly with depth throughout the entire layer have been published by several investigators (e.g., Gibson 1967; Carrier and Christian 1973). These solutions are of considerable value; however, application of these solutions to determining rock modulus from a plate load test is limited by the fact that firstly, one cannot take account of the effect of a disturbed or jointed rock overlying intact rock, and secondly, no general solutions for the variation in displacement with depth below the footing have been given. A limited range of solutions for the surface settlement of a plate on a two layer system (assuming each layer to be homogeneous) has been published by Manfredini et al. 1974.

In this paper, elastic solutions will be presented for a plate or footing resting on an isotropic rock mass with a modulus profile as shown in Fig. 1. Influence factors will be given for determining the displacement both beneath and outside the loaded region. Two procedures will also be described for determining the rock mass modulus from plate load test results. The first procedure will use the results obtained from three plate load tests, using plates of different diameters, for predicting the rock modulus variation. The second procedure will use the measured variation in displacement with depth below the plate to estimate the rock modulus variation from the measured modulus, $E_k$, at some depth, $z_k$, and the measured changes in stiffness with depth, $\rho$, giving $E_0 = E_k - \rho z_k$, provided that $z_k < z_c$.

Unless otherwise noted, it is assumed that a circular plate applies a uniform pressure to the rock mass.

### Solutions for surface displacement

The settlement, $S_x$, at any point, $x$, on the rock surface due to a uniform pressure $q$ acting on an area of diameter $B$ may be expressed in the form

$$ S_x = qB(1 - v^2)I_x/E_0 $$

where $I_x$ is the influence factor for the settlement at the position $x$ and is a function of the dimensionless parameters $z/\sqrt{B}$ and $\rho B/E_0$, and all other terms are as defined above.

The influence factor determined at the centre line of the plate, $I_c$, the edge of the plate, $I_e$, and at distances of $0.75B$ and $1.5B$ from the centre of the plate are given for Poisson’s ratio $v = 0.3$ in Figs. 2–5 respectively. These figures show the influence factor as a function of the weathered or jointed depth, $z/\sqrt{B}$, for a range of increases in modulus with depth, $\rho B/E_0$. For a circular plate, the rock properties to a depth of approximately two diameters largely govern the settlement of the plate and, consequently, results are only given for $z/\sqrt{B}$ less than three. The variation in centre settlement factor, $I_c$, as a function of $\rho B/E_0$ for a range of depths, $z/\sqrt{B}$, is shown in Fig. 6.

The presence of a weathered or jointed rock zone above intact rock influences both the magnitude and the distribution of settlement. The effect of this nonhomogeneity is best appreciated by considering the normalized surface displacement profiles shown in Fig. 7. Nonhomogeneity appreciably increases the settlement below the loaded area (relative to what would be obtained for unweathered intact rock); however, the

### Principal assumptions

In this paper it is assumed that the rock mass may be idealized as a deep, isotropic elastic continuum with Poisson’s ratio, $v$, and a Young’s modulus, $E(z)$, at any depth, $z$, which is given by the equations:

1a. $E(z) = E_0 + \rho z$,  $z \leq z_c$

1b. $E(z) = E_0 + \rho z_c$,  $z > z_c$

where $E_0$ is Young’s modulus at the rock surface, $\rho$ is the rate of increase in modulus with depth, and $z_c$ is the depth of the weathered or fractured region.

It is convenient to non-dimensionalise the increase in stiffness with depth with respect to the surface modulus $E_0$ just below the plate. Thus for a given plate diameter $B$, the modulus profile described above, and shown in Fig. 1, may be characterised by: the dimensionless depth of the weathered zone, $z_c/B$; and the dimensionless increase in stiffness with depth, $\rho B/E_0$.

It is recognised that in practice it may sometimes be difficult to determine the surface modulus, $E_0$, directly; however, in these cases the value of $E_0$ may be estimated from the measured modulus, $E_k$, at some depth, $z_k$, and the measured changes in stiffness with depth, $\rho$, giving $E_0 = E_k - \rho z_k$, provided that $z_k < z_c$.

In this paper, elastic solutions will be presented for a plate or footing resting on an isotropic rock mass with a modulus profile as shown in Fig. 1. Influence factors will be given for determining the displacement both beneath and outside the loaded region. Two procedures will also be described for determining the rock mass modulus from plate load test results. The first procedure will use the results obtained from three plate load tests, using plates of different diameters, for predicting the rock modulus variation. The second procedure will use the measured variation in displacement with depth below the footing have been given. A limited range of solutions for the surface settlement of a plate on a two layer system (assuming each layer to be homogeneous) has been published by Manfredini et al. 1974.

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Unless otherwise noted, it is assumed that a circular plate applies a uniform pressure to the rock mass.
settlement of points outside the footing is only slightly affected. The sensitivity of settlement to nonhomogeneity decreases with increasing distance from the centre line of the loaded area. This can be easily demonstrated using the results presented in Figs. 2–5 and is illustrated for a number of cases in Table 1.

The relative insensitivity of settlement outside the loaded area to nonhomogeneity creates considerable
practical difficulties in attempting to infer the modulus variation with depth from surface settlement observations due to a single plate test. These difficulties arise from the fact that even small errors in the measurement of the small displacements outside the footing could significantly alter the interpretation of the modulus variation with depth.

The results presented in Figs. 2–7 were obtained for Poisson's ratio \( \nu = 0.3 \). The majority of the effect of Poisson's ratio is taken into account by the term \((1 - \nu^2)\) in [2]; however, for a nonhomogeneous rock, the influence factor, \( I_c \), will also be a function of Poisson's ratio. Poisson's ratio for rock typically lies between 0.3 and 0.1. The corresponding variation in influence factor, \( I_c \), is less than 10% for a wide range of nonhomogeneous profiles and is less than 5% for moderate levels of nonhomogeneity \( (z_e/B < 2, \rho B/E_0 < 1.5) \). For most practical applications, this secondary effect of Poisson's ratio may be ignored.

*Settlement of a rigid plate*

The foregoing results were for settlement due to a perfectly flexible plate. To sufficient accuracy, the displacement of a rigid plate or footing can be taken to be the average displacement of a footing that causes a vertical stress increase given by the expression

\[
\sigma(x) = \frac{P}{\pi B \left[ \left( \frac{B}{2} \right)^2 - x^2 \right]^{1/2}}, \quad 0 < x < \frac{B}{2}
\]

where \( P \) is the magnitude of the applied load, and \( x \) is the distance from the centre of the footing.

This approach allows considerable computational savings and for a wide range of nonhomogeneity parameters was found to give results within better than 3% of those obtained by more rigorous methods.

The settlement of a rigid footing, calculated as described above, is given in Figs. 8 and 9 for a range of values of \( z_c/B \) and \( \rho B/E_0 (\nu = 0.3) \). These results, and the results given in Figs. 2 and 3 may be used for estimating displacements for the limiting cases of a rigid and a uniformly loaded plate or footing. Typically, rigidity of the plate will reduce the maximum settlement by up to 25%.

*Prediction of subsurface modulus variation using a series of surface plate load tests*

It is well established that any field testing programme
Table 1. Effect of nonhomogeneity upon settlement of points on the rock surface

<table>
<thead>
<tr>
<th>Parameters defining nonhomogeneity</th>
<th>Settlement due to loading on weathered rock mass</th>
<th>Settlement due to loading on intact (homogeneous) rock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal location relative to centre line of loading</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Centre</td>
<td>Edge (0.5B)</td>
</tr>
<tr>
<td>$z_e/B$</td>
<td>$pB/E_0$</td>
<td>$E_d/E_B^*$</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.667</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>1.19</td>
</tr>
<tr>
<td>10</td>
<td>0.167</td>
<td>1.53</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.333</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>2.45</td>
</tr>
<tr>
<td>10</td>
<td>0.048</td>
<td>4.39</td>
</tr>
</tbody>
</table>

$*E_B$ (homogeneous) = $E$ (below weathered or jointed region) = $E_0 + p z_e$.  

Fig. 8. Variation in average settlement factor $I$ with $z_e/B$.  

Fig. 9. Variation in average settlement factor $I$ with nonhomogeneity $pB/E_0$.  

should be supported by an appropriate subsurface investigation. If such a subsurface investigation indicates the presence of a rock profile consistent with a regular increase in stiffness with depth, then the relevant field parameters may be estimated either from the settlement of a number of different size plates, or alternatively, from the settlement with depth below a single plate. These two approaches will be discussed in this and the following sections.

Three parameters (viz., $E_0$, $p$, and $z_e$) are required to define a rock modulus profile such as shown in Fig. 1. For the determination of these three parameters from plate settlement observations, it is necessary to perform three plate load tests using plates of different diameters.

(As previously noted, the number of tests could be reduced by using settlement observations outside the plate, but this is not recommended.) If the subsurface investigation indicates that the increase in modulus extends to considerable depth then the rock profile assumes the form of a Gibson soil and only two tests are required to determine the parameters $E_0$ and $p$ ($z_e \rightarrow \infty$).

Previous investigators (e.g., Davis and Taylor 1961) have shown the average settlement of a perfectly flexible plate on a deep homogeneous deposit is approximately equal to the average of the centre and edge settlement of the perfectly flexible plate, and to the settlement of a rigid plate, to an accuracy of ±5%. The results
presented in Figs. 2, 3, and 8 show that the same approximation can also be used for the range of nonhomogeneity considered herein to an accuracy of better than ±5%. Thus to provide the greatest generality regarding the use of the solutions presented in this section, the modulus variation will be determined in terms of the average settlement of the plate. The use of this quantity largely eliminates the effect of plate flexibility upon the deduction of the modulus profile.

Two plate test \((z_c \rightarrow \infty)\)

Assuming initially that \(z_c\) approaches infinity, the average settlement of two plates of diameters \(B_1\) and \(B_2\) respectively are given by the expressions

\[
S_1 = q_1 B_1 (1 - \nu^2) I_1 / E_0 \\
S_2 = q_2 B_2 (1 - \nu^2) I_2 / E_0
\]

where \(q_1\) and \(q_2\) are the applied pressures, and \(I_1\) and \(I_2\) are influence factors which are functions of \(pB_i/E_0\) and \(pB_j/E_0\) respectively.

The plate sizes \(B_1\) and \(B_2\) and the applied pressures \(q_1\) and \(q_2\) are specified and the average settlements \(S_1\) and \(S_2\) are measured. Thus the ratio of the influence factors, \(I_2/I_1\), may be determined from [4], viz.,

\[
\frac{I_2}{I_1} = \frac{S_2 q_1 B_1}{S_1 q_2 B_2}
\]

For a given ratio of plate sizes \(B_2/B_1\), the variation in the ratio \(I_2/I_1\) can be theoretically determined as a function of the dimensionless nonhomogeneity \(pB_i/E_0\) as shown in Fig. 10. Thus knowing \(I_2/I_1\) from [5], the value of \(pB_i/E_0 = C\) (say) may be deduced from Fig. 10 for \(B_2/B_1 = 2\), or from similar charts which are readily compiled for \(B_2/B_1 = 2\).

Knowing \(pB_i/E_0 \neq C\), from above, \(I_1\) may be determined from Fig. 9. Assuming a value of Poisson’s ratio, \(\nu\), \(E_0\) can then be calculated from [4], viz.,

\[
E_0 = q_1 B_1 (1 - \nu^2) I_1 / S_1
\]

and hence,

\[
q = CE_0 / B_1
\]

where [6] defines the rock profile.

Three plate test

If the subsurface investigation suggests that the rock is likely to have a relatively constant modulus below some small depth, \(z_c\), then it is necessary to perform three tests for plates with diameters \(B_1\), \(B_2\), and \(B_3\) giving settlements \(S_1\), \(S_2\), and \(S_3\). Thus,

\[
S_i = q_i B_i (1 - \nu^2) I_i / E_0
\]

where \(i = 1, 2, 3\), and \(B_2 < B_1 < B_3\).

In this case two influence factor ratios must be determined, viz.,

\[
\frac{I_2}{I_1} = \frac{S_2 q_1 B_1}{S_1 q_2 B_2} \\
\frac{I_3}{I_1} = \frac{S_3 q_1 B_1}{S_1 q_3 B_3}
\]

For a given set of plate size ratios, \(B_2/B_1\) and \(B_3/B_1\), the ratios of \(I_2/I_1\) and \(I_3/I_1\) may be related to the dimensionless parameters, \(z_c/B_1\) and \(pB_i/E_0\), as shown in Fig. 11 for the case where \(B_2/B_1 = 0.5\) and \(B_3/B_1 = 2\) (similar charts for other ratios of \(B_1, B_2,\) and \(B_3\) may be generated from the results given in Figs. 8 and 9).

Using the values of \(I_2/I_1\) and \(I_3/I_1\) determined from [8] in conjunction with Fig. 11, the dimensionless parameters, \(z_c/B_1\) and \(pB_i/E_0\), may be deduced. Thus the value of \(I_1\) may be determined from Fig. 8 or 9 and the values of \(E_0\), \(p\), and \(z_c\) calculated from [6a, b] and the value of \(z_c/B_1\) respectively. This procedure is illustrated by a worked example in the appendix.

The procedure described above is in terms of the average or rigid settlement of a plate. However, if the centre settlement of a very flexible plate is the only quantity measured, then a similar procedure may be adopted using Figs. 10 or 11 to determine the dimensionless parameters, but using Fig. 2 to determine the theoretical value of \(I_1\) for use in [6]. This procedure is not exact but should be sufficiently accurate for most practical purposes (spot checks indicate that the ratios of \(I_2/I_1\) etc., obtained from central settlement and rigid settlements respectively, typically differ by less than 0.5%).

The preceding development has also assumed that the entire area of the plate is loaded. A similar procedure

![Fig. 10: Settlement ratios \(I_2/I_1\) for a deep weathered layer \((z_c/B = \infty)\).](image-url)
may be adopted for a circular flat jack with a central hole, provided that the effect of the unloaded region is considered. Since the rock is assumed to be elastic, provision for the hole can readily be made by application of the principle of superposition.

The cost of performing three plate load tests using plates of different diameters can often be justified on large projects; however, the above procedure may not be suitable on some projects because of cost or because a suitable reference point for measuring displacements is not available. In these cases it may be necessary to adopt an alternative approach as described in the next section.

**Prediction of subsurface modulus variation from the variation in displacement with depth**

The displacement profile with depth beneath a loaded plate can be monitored by using a circular flat jack with a small hole in the centre, with an instrumented borehole that extends to a depth of at least six plate diameters, located directly beneath the centre of the flat jack. The displacements up to a depth of 3 flat jack diameters will generally be measured with respect to the anchor located at a depth of 6 diameters. Since the displacement at a depth of 6B is small, this procedure allows the determination of displacements with respect to a relatively stable reference point, although some movement of the latter will occur.

The relative settlements of different points beneath the plate will depend upon the modulus profile. Since varying either \( \frac{z}{B}\) or \( \frac{pB}{E_0}\) will alter the displacement profile in different ways, the actual shape of the displacement profile can be used to identify the modulus profile.

The presence of the hole may be expected to have some effect upon the displacement profile. This effect is greatest directly beneath the plate and decreases rapidly with increasing lateral or vertical distance from the plate. It can be easily demonstrated that the hole in the plate will have relatively little effect upon displacements at depths of more than one hole diameter. In determining the settlements due to this plate, the pressure \( q \) may be reduced to correspond to the actual applied load, however for typical hole dimensions this correction is only a few percent.

Suppose that all displacements are measured with respect to the anchor at a depth of 6B, then the measured apparent displacement, \( \delta_{z*} \), at a depth, \( z \), is

\[ \delta_{z*} = \delta_z - \delta_{6B} \]
where $\delta_z$ and $\delta_{6B}$ are the true displacements at depths $z$ and $6B$ respectively. To allow convenient determination of the rock parameters it is necessary to normalize the displacement profile with respect to the displacement at some given depth. In this paper it is suggested that the displacements be normalized with respect to the average apparent displacements at a depth of $0.25B$ and $0.5B$, thus:

$$\delta_{nP} = (\delta_{0.25B} + \delta_{0.5B})/2$$

and the normalized displacement, $I_z^*$, at any depth, $z$,

is:

$$I_z^* = \frac{\delta_z^*}{\delta_{nP}^*}$$

(The term $\delta$, corresponding to settlement below the surface, is used so as to distinguish these settlements from settlements, $S$, at the rock surface.)

The normalizing depths of $z = 0.25B$ and $0.5B$ were selected because they are in a region of relatively high stress but are sufficiently deep to avoid complications due to the hole in the plate. The normalizing displacement was taken as the average of these two displace-
ments so as to minimize the effects of any error in the measurement of either of these quantities upon the normalized displacement profile.

Thus the measured displacements, $\delta z^*$, obtained from a plate load test should be normalized with respect to $\delta_n^*$ and plotted as a function of dimensionless depth at a scale corresponding to that of Figs. 12–15. These figures give the theoretical normalized displacement profiles for a range of depths, $z/B$, and nonhomogeneities, $\rho B/E_0$ (to sufficient accuracy the curves for $z/B = 5$ may also be used for values of $z/B > 5$). Increasing the nonhomogeneity, $\rho B/E_0$, tends to increase the normalized displacement near the surface and decrease it at depth. This trend is enhanced by increasing the depth, $z/B$. By overlaying the plotted normalized field displacement profile upon Figs. 12–15 and selecting the case which best approximates the observed profile, an estimate can be made of the dimensionless parameters, $z/B$ and $\rho B/E_0$. Using these parameters, the dimensionless displacement factors, $I_{0.25B}$, $I_{0.5B}$, and $I_{6B}$, at depths of $z = 0.25B$, $0.5B$, and $6B$ may be obtained from Figs. 16 and 17; thus

Fig. 13. Normalized settlement variation with depth, $z/B = 1$. 
Fig. 14. Normalized settlement variation with depth, \( z/B = 2 \).

\[ \delta_{0.25B}^* = qB(1 - \nu^2)[I_{0.25B} - I_0]/E_0 \]

Substituting the observed apparent displacements, \( \delta_{0.25B}^* \), into [12] and estimating a value of Poisson’s ratio, \( \nu \), permits the value of \( E_0 \) to be calculated. The values of \( z_c \) and \( \rho \) can then be directly determined since \( z_c/B, \rho B/E_0, B, \) and \( E_0 \) are now all known.

Similarly, the observed apparent displacement \( \delta_{0.5B}^* \) could be substituted into the equation:

\[ \delta_{0.5B}^* = qB(1 - \nu^2)[I_{0.5B} - I_0]/E_0 \]

and the values of \( E_0, \rho, \) and \( z_c \) calculated as described above. In theory, the values of \( E_0 \) obtained from [12] and [13] should be identical. Any difference between the two calculated values may be largely considered to be an indication of the error involved in the determination of the modulus profile.

**Application to a field case**

The procedures described in the previous two sections may be used for estimating the modulus variation for
rock which can reasonably be expected to have a profile such as shown in Fig. 1. This judgement can often be made on the basis of borehole logs and previous experience. Clearly in applying these concepts, consideration must be given to scale effects with regard to plate size, dominant joint spacing, and the size and type of geotechnical structure to be constructed.

The application of the two-three plate procedure is illustrated by a worked example in the appendix. In this section, consideration will be given to the application of the second procedure to the prediction of the variation in modulus with depth in Mudford chalk.

A well-documented geotechnical investigation and case history involving the settlement of a large steel tank on chalk at Mudford, Norfolk has been described by Ward et al. (1968) and the modulus profile has been interpreted in some detail by Burland et al. (1978). Briefly, a 18.3 m diameter tank was constructed on chalk that varied gradually from a highly weathered material at the surface to intact unweathered rock at
depth. Two thin marl seams were located at depths of 8.4 m and 22.9 m. When filled, the tank exerted a pressure of 180 kPa. The vertical deflections beneath and adjacent to the tank were measured using transducers anchored to the walls of 0.9 m diameter shafts bored to a depth of 16.3 m below the tank. Deflections at greater depth were measured relative to the bottom of the shaft.

Numerous settlement measurements were taken over a range of depths; however, none of these depths correspond to the dimensionless depths, $\frac{z}{B}$, of 0.25 and 0.5 used in Figs. 12–15. One solution to this problem would be to construct a curve of best fit through the observed data points and then estimate the settlement at the appropriate dimensionless depths. This procedure could be readily adopted in practice although it would generally be preferable to locate settlement measurement points at depths of 0.25B and 0.5B whenever possible. A more accurate but, in the absence of a computer program, more inconvenient approach would be to develop settlement profiles normalized with respect to the appropriate depths for a particular case. In this instance settlements were measured at depths $z = 0.2B$ and $0.4B$ ($B = 18.3$ m). Displacement profiles normalized with respect to the average of the settlements at these depths could be constructed from the data presented in this paper (although this is cumbersome and is not recommended) or directly from the computer program FLANS (Rowe and Booker 1980). For the purposes of this paper, the latter procedure was adopted and displacement profiles (normalized with respect to $\delta_n = \frac{(\delta_{0.2B} + \delta_{0.4B})}{2}$) were obtained for a range of disturbed depths, $\frac{z}{B}$, and nonhomogeneties, $\frac{pB}{E_0}$. (These results were actually obtained for $\nu = 0.24$ which corresponds to the value reported by Burland et al. (1978) although the difference between the normalized curves for $\nu = 0.24$ and 0.3 is small.)
The majority of combinations of $z/B$ and $pB/E_0$ could be dismissed immediately upon comparison with the normalized observed data points shown in Fig. 18. A limited number of the theoretical profiles are shown in Fig. 18 to illustrate the matching procedure. The curves for $pB/E_0 = 25$, $z/B = 1.5$ and $pB/E_0 = 5$, $z/B = 2$ give the best agreement with the observed data points and are very similar below a depth of $0.25B$. The latter case roughly corresponds to the modulus profile deduced from finite element results by Burland et al. (1978) and gives a good match everywhere but at the surface. The curve for $pB/E_0 = 25$ and $z/B = 1.5$ provides an equally good match below $0.25B$ but also gives a good match with observed displacement at the surface and on this basis would appear to be a more appropriate modulus profile.

Assuming that the modulus profile is defined by the dimensionless parameters $z/B = 1.5$ and $pB/E_0 = 25$ as suggested by the matching procedure described above, the parameters $E_0$ and $p$ may be deduced by collocation of the observed and theoretical displacements at a depth of $0.2B$ or $0.4B$; thus at $0.2B$, 

$$\delta_{0.2B} = \frac{qB(1 - \nu^2)l_{0.2B}}{E_0}$$

where $l_{0.2B} = 0.0555$ for $z/B = 1.5$, $pB/E_0 = 25$, $q = 0.18 \text{ MPa}$, $B = 18.3 \text{ m}$, $\nu = 0.24$, and $\delta_{0.2B} = 1.19 \text{ mm}$. Thus $E_0 = 0.18 \times 18.3 \times (1 - 0.24^2) \times 0.0555/(1.19 \times 10^{-3}) = 0.145 \times 10^3 \text{ MPa}$, $\rho = 25E_0/B = 198 \text{ MPa/m}$, and $z_c = 1.5B = 27.5 \text{ m}$.

Similarly collocation of the observed and theoretical displacement at $0.4B$ gives $E_0 = 0.132 \times 10^3 \text{ MPa}$ and $\rho = 181 \text{ MPa/m}$. The discrepancy between these modulus parameters provides some indication of the error involved with the data. In particular, the presence of the thin marl seam very close to the point at $0.4B$ may be responsible for slightly inconsistent behaviour at this point and so the modulus parameters obtained for collocation at $0.2B$ will be adopted as the relevant parameters for this rock mass.

To test the validity of this deduced modulus profile, it has been plotted in Fig. 19a along with the modulus values deduced by Burland et al. (1978) from the plate load tests performed at different depths by Ward et al. (1968). Although the plate load test data show considerable scatter, the deduced modulus profile appears to be generally consistent with them.

The theoretical displacement profiles obtained at the centre and edge of the tank using this deduced modulus profile are compared with the corresponding observed settlements in Fig. 19b. As might be expected, the centre line settlements are in excellent agreement except for two data points directly above the two marl seams. In comparing the edge settlement it is to be remembered that the presence of a 0.9 m diameter shaft is likely to have some effect upon the observed settlements and hence the quite good agreement between the observed and predicted settlements at the edge provides additional support for the validity of the deduced modulus profile.

This example shows that the proposed procedure of matching theoretical and observed normalized displacement profiles provides a convenient means of obtaining
the rock modulus profile, at least for some rocks. Furthermore, this deduced profile has been shown to be consistent with an independent determination of modulus profile using plate load tests at different depths as well as providing a good prediction of settlements near the edge of the tank.

Conclusions

Weathering, or the variation in the frequency and tightness of joints, may result in an increase in mass modulus with depth for some rocks. This increase in modulus will continue until a depth is reached at which the rock behaves as a sound, intact unit and the modulus will be relatively constant with depth below this point. In this paper, elastic solutions have been presented for the deformation of such a rock mass due to a uniform circular loading and an approximately rigid circular loading.

Two procedures have been described for determining the rock mass modulus from plate load test results. The first procedure used the results from three plate load tests (the plates having different diameters) to infer the variation in mass modulus with depth. The second procedure used the measured variation in displacement with depth below a single plate to infer the mass modulus variation.

The application of the two procedures has been illustrated by a worked example (in the appendix) and by consideration of a field case where the inferred modulus could be compared with alternative modulus variation data.

As with all geotechnical analysis, the results presented in this paper must be used in conjunction with good engineering judgment. Under these circumstances, it is considered that the procedures described in this paper should provide a convenient and economical tool for determining rock modulus parameters for the selected class of rock conditions described above.

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Appendix: worked example

The problem

Three plate load tests were performed using plates with diameters of 0.3, 0.6, and 1.2 m. For an applied pressure of 400 kPa, the observed average settlements were 0.32, 0.56, and 0.97 mm respectively. Using these data, and assuming that the borehole logs indicate a rock whose modulus profile might be consistent with that shown in Fig. 1, calculate the modulus profile parameters \(E_0\), \(\rho\), \(z_c\) and the modulus below the disturbed region, \(E_h\).

Solution

Let \(B_1 = 0.6\) m, \(B_2 = 0.3\) m, and \(B_3 = 1.2\) m. Thus \(S_1 = 0.56\) mm, \(S_2 = 0.32\) mm, \(S_3 = 0.97\) mm, \(B_2/B_1 = 0.5\), and \(B_3/B_1 = 2.0\). From [8],

\[
\frac{I_2}{I_1} = \frac{S_2 q_1 B_1}{S_1 q_2 B_2} = \frac{0.32 \times 400 \times 0.6}{0.56 \times 400 \times 0.3} = 1.14
\]

and

\[
\frac{I_3}{I_1} = \frac{S_3 q_1 B_1}{S_1 q_3 B_3} = \frac{0.97 \times 400 \times 0.6}{0.56 \times 400 \times 1.2} = 0.866
\]

For these two dimensionless ratios, Fig. 11 may be used to determine \(z_c/B_1 = 1.94\) and \(\rho B_1/E_0 = 0.78\).
For \( \rho B_1/E_0 = 0.78 \) and \( z_c/B_1 = 1.94 \), Fig. 8 gives a value of \( I_1 = 0.51 \).

Assuming \( v = 0.25 \), [6a] then gives

\[
E_0 = \frac{q_1 B_1 (1 - v^2) I_1}{S_1} = \frac{0.4 \times 0.6 \times (1 - 0.25^2) \times 0.51}{0.56 \times 10^{-3}} \approx 0.2 \text{ GPa}
\]

From \( \rho B_1/E_0 = 0.78 \) and [6b], \( \rho = 0.78 \times 0.2/0.6 = 0.26 \text{ GPa/m} \), and \( z_c = 1.94 B_1 = 1.94 \times 0.6 = 1.16 \text{ m} \).

Therefore the intact modulus below the disturbed region should be

\[
E_h = E_0 + \rho z_c = 0.2 + 0.26 \times 1.16 = 0.5 \text{ GPa}
\]