Stabilization of Very Soft Soils Using High Strength Geosynthetics: the Role of Finite Element Analyses

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ABSTRACT

The application of finite element (FE) techniques for the analysis of reinforced embankment behaviour is reviewed. Details such as the choice of finite element and constitutive models as well as the validation of finite element results against benchmark solutions are discussed.

Results from the authors’ finite element analyses are examined, and it is shown that FE analysis can be particularly useful for identifying the mechanisms of failure and also for indicating why a geosynthetic reinforcing material may substantially improve stability for a certain foundation strength profile whereas for different foundation strength profiles the same reinforcement may give rise to negligible improvement in embankment stability.

The use of plasticity solutions for estimating the maximum effect of reinforcement is illustrated. The results of finite element analyses are then used to demonstrate that although the collapse load calculated from plasticity theory can be attained for very highly reinforced embankments, in many situations failure will occur at embankment heights well below the collapse height. It is then demonstrated that the failure height for a reinforced embankment is related to the modulus of the reinforcement but is not very sensitive to the modulus of the soil.

The development of strain in the geotextile is examined and it is demonstrated both from field evidence and theoretical analysis that the reinforcement plays a relatively small role at low load levels since the soil is essentially elastic. Significant strain in the geotextile begins to develop with increasing plasticity and in fact most of the strain is developed after a contiguous plastic...
region is developed in the soil, since beyond this point the reinforcement is all that prevents collapse from occurring. As a consequence, the strains developed in reinforcement for a given embankment height will largely depend on the height of the embankment relative to the height at which contiguous plasticity occurs, and hence will be sensitive to the magnitude and distribution of the actual shear strength in a soil deposit.

1 INTRODUCTION

It has now been well established that the provision of geosynthetic reinforcement within an embankment or dike constructed on very soft soils can substantially improve stability and allow construction to heights considerably in excess of that which would be practicable without reinforcement (e.g. see Refs 1–4). Numerous simple design methods based on consideration of limit equilibrium have been proposed (e.g. see Refs 5–7). An indication of the maximum improvement in performance which can be achieved by reinforcement can also be obtained from classical plasticity bearing capacity solutions.

Neither the limit equilibrium analyses nor the plasticity solutions provide information concerning the embankment deformations and reinforcement strains which may be associated with a given reinforcing scheme. In reality, of course, reinforced embankments are a composite system involving at least three components: namely, the foundation, the embankment fill and the reinforcement. The performance of the reinforced embankment will depend on the interaction between these components and to a large extent, this interaction will arise from strain compatibility requirements at the interface between the various components. This then raises the question as to how important is consideration of interaction and to what extent will the performance of the reinforced system depend on deformations and strains in the reinforcement. A second, but related, question concerns the magnitude of the strains and deformations that are to be expected under working conditions.

These questions could all be answered by the construction and monitoring of a large number of full scale field test embankments. Unfortunately, the cost of performing and adequately monitoring a sufficiently large number of full scale embankments is so large that this is not practical. Finite element techniques provide a cost effective alternative to construction of a large number of test embankments by allowing us to perform numerical simulations of embankment construction for a wide range of different situations with the objective of providing a ‘data base’ which can be used to validate
approximate methods of analysis and to indicate the conditions under which these analyses are applicable.

Finite element techniques have the potential to allow us to:

- improve our understanding of observed behaviour in field trials;
- model the complete response of a reinforced embankment up to collapse;
- examine the effects of changes in the elements of the system (i.e. the properties of the reinforcement and the soil); and
- investigate changes in construction procedures and the nature of the system.

The finite element technique is well recognized as being a very powerful tool and numerous examples of its application to modelling of reinforced embankments can be found in the literature (e.g. see Refs 2, 4, 8–18). However, the use of finite element analyses for performing studies which could answer the questions raised above is subject to some important constraints.

If one is to model the response of the embankment up to collapse, then it is essential to adopt a finite element formulation and constitutive model which (a) models the stress-dependent properties of the fill material; (b) correctly models plastic failure and plastic flow in the fill and the foundation; and (c) allows for potential slip at the soil–reinforcement interface.

It should also be recognized that there are many different finite element models with different characteristics and capabilities. The only way to determine if a particular formulation and program is likely to give reasonable results is to ‘validate’ the program against (a) limiting analytical benchmark solutions; (b) available data relating to the construction and performance of full scale embankments; and/or (c) centrifuge test data.

Finally, it is noted that although a finite element study may be a cost-effective alternative to a full scale field study, it is not without cost. A good elasto-plastic FE analysis requires an experienced ‘driver’ and considerable data preparation time.

The primary roles of finite element analysis are (a) for research into the behaviour of reinforced embankments including validation of simplified methods of analysis; and (b) for complementing conventional analyses on large/important projects or where the anticipated conditions are such that the validity of simplified approaches may be questioned.

The objective of this present paper is to review some of the important factors to be considered in the use of the finite element technique to analyze reinforced embankments and to discuss some of the authors’ findings which have resulted from the use of the finite element technique in the analysis of reinforced embankments.
2 SELECTION OF FINITE ELEMENT AND CONSTITUTIVE MODEL

Modelling of the discrete components of a reinforced soil system (i.e. the soil, the reinforcement and the soil–reinforcement interface) involves consideration of both the type of finite element and the constitutive relationship that will be adopted. This is particularly so if the analysis is intended to be carried to collapse.

2.1 The soil

There are many different types of finite elements which could potentially be used to represent the soil. Restricting attention to plane strain conditions, it has been shown\textsuperscript{19,20} that constant strain, linear strain, quadratic strain and cubic strain triangles can all give accurate predictions of undrained collapse for soft foundations, although if constant strain triangles are used, they must be used in a crossed triangle formation (i.e. rectangles are divided into four isosceles triangles—see Ref. 19).

A number of quadrilateral elements could also be used but care is needed in the choice of element and integration rule. The most commonly used quadrilateral elements have either four nodes or eight nodes (see Ref. 21). The four noded element is not suitable for predicting collapse loads.\textsuperscript{20} The eight noded element is most commonly used in conjunction with a reduced (2 \times 2) integration rule suggested by Zienkiewicz.\textsuperscript{21} This approach has the beneficial effect of avoiding the phenomenon of ‘locking’ and potentially improving the calculation of collapse loads while decreasing the cost of the computation (as compared with an analysis using full 3 \times 3 integration). However, as indicated by Nagtegaal and De Jong\textsuperscript{22} and Sloan,\textsuperscript{23} the use of reduced (2 \times 2) integration with eight noded quadrilateral elements can give rise to unrealistic deformation patterns and care is required to ensure satisfactory results are obtained.

The choice of which ‘suitable’ element to use will depend on consideration of efficiency and convenience. If the concern is simply to determine the bearing capacity factors for rigid footings on clay, then it has been demonstrated\textsuperscript{20,24} that the high order (e.g. 15 noded cubic strain) triangles are the most efficient, in that they allow the determination of accurate collapse loads with a minimum number of nodes and elements. However, in the analysis of reinforced embankments, the size of elements (particularly in the fill) will be dictated by constraints related to simulation of the construction process and geometry, and in these cases the potential gain in efficiency may be lost because of the large number of degrees of freedom in the fill that are required to model construction in small fill lifts. Thus the choice of element
ceases to be a clear-cut decision, and although the present authors' work has indicated that there are computational advantages to using cubic strain triangles for the analysis of bearing capacity problems where shear strength increases with depth, for analyses of reinforced embankments on these same foundations it has been found to be convenient and efficient to use low order (constant strain) triangles for modelling the soil in the foundation and fill.

Constitutive models for the soil may be subdivided into two categories namely, non-linear elastic and (non-linear) elasto-plastic. Non-linear elastic (e.g. hyperbolic) models can be expected to provide acceptable results at low stress levels (e.g. when there is a large 'factor of safety'); however, since they are based on elastic theory, they cannot correctly model plastic failure and plastic strains within the soil mass (it is noted that the use of a cohesion intercept c and friction angle \( \phi \) in a hyperbolic model does not imply that the model is a plasticity model---e.g. see Ref. 25). Consequently, these models are not suitable for calculating collapse heights.

Numerous plasticity formulations have been proposed in the literature. The simplest of these involves a Mohr–Coulomb failure surface and a non-associated flow rule.\(^{26}\) This model has been successfully applied in the analysis of geotextile reinforced embankments.\(^{2,10,12,27}\) This form of analysis can be readily modified to include the consideration of a non-linear failure envelope commonly encountered with granular materials.\(^{28}\) These models can be expected to model the soil behaviour up to and including failure. By examining the results of studies performed using this class of model it is possible to assess the magnitude of the strains to be expected prior to collapse of the structure and hence to make some initial assessment of potential significance of strain softening. However, this class of model is not suitable for modelling strain-softening behaviour, and indeed the modelling of localization and strain softening even for unreinforced materials requires considerable additional research.

In modelling the behaviour of the embankment, it is essential to consider the variation in soil stiffness with increasing stress level during construction, since this can have a significant influence on the stresses and displacements developed within the reinforced embankment. The simplest way of modelling this is to adopt a non-linearity based on Janbu's equation, viz.

\[
\frac{E}{P_s} = K\left(\frac{\sigma}{P_a}\right)^m
\]

where \( E \) is the Young's modulus of the soil, \( \sigma \) is the minor principal stress or the mean stress depending on the details of the formulation, \( P_a \) is atmospheric pressure and \( K \) and \( m \) are the material parameters. This non-linearity is included in the 'hyperbolic' model and can also be readily included in elastic–plastic models. It should be recognized that the modelling of 'yield'
implicit in eqn (1) is only approximate and is not appropriate for situations where there may be cyclic loading. However, there is considerable evidence to suggest that this approach can provide reasonable results for problems involving monotonic loading (as is generally the case in modelling embankment construction).

2.2 The reinforcement

The reinforcement can be modelled using a one-dimensional bar element. Non-linearity of the stress strain behaviour and yield can also be readily modelled by making the element stiffness a function of stress (or strain) level. Breakage (snap) of the reinforcement can also be modelled; however, this involves the redistribution of stresses developed in the reinforcement prior to breaking, and erroneous stress distributions can be obtained unless particular care is taken with the numerical algorithm used to redistribute these stresses.

2.3 The soil–reinforcement interface

The interaction between the soil mass and the reinforcement can be modelled by introducing soil–reinforcement interface elements. This can be achieved in a number of ways, including the use of joint elements, nodal-compatibility slip elements or by substructuring. Common approaches to modelling the soil–reinforcement interface involve three nodes at each point along the reinforcement; one attached to the soil above the reinforcement, one on the reinforcement, and one attached to the soil below the reinforcement. The nodal-compatibility slip element (which may be formulated initially in terms of normal and tangential springs with very high stiffnesses) (a) ensures compatible displacement between a pair of dual nodes (one attached to the soil and one attached to the reinforcement) until a Mohr–Coulomb failure criterion is reached, and (b) replaces the compatibility conditions by a failure condition and dilatancy equation once the interface strength is exceeded. Joint elements allow relative deformation of the soil and reinforcement, prior to failure of the interface, based on some assumed constitutive relationship of what is in effect an interface layer between the reinforcement itself and the general soil continuum (e.g. Andrawes et al. used a hyperbolic model to represent the interface behaviour).

In its simplest form, the joint element may be comprised of a pair of normal and tangential springs. Clearly, as the stiffness of a joint element increases, it tends to a nodal-compatibility slip element, and the distinction between the two is related to the question of whether a distinct interface layer exists or whether the deformations at the interface (prior to failure) are
simply due to the interaction between the reinforcement and the soil on either side of the interface. If there are good experimental data indicating that a distinct interface layer exists with experimentally defined stress-strain characteristics, then this can be readily modelled as a joint element or as a thin layer of continuum element (with slip still being modelled using a nodal-compatible slip element). In the absence of this data, a nodal-compatibility slip element would seem appropriate.

Any modelling of interface behaviour must consider three possible mechanisms of failure as noted below.

(a) If there is insufficient anchorage capacity, failure will occur at the soil reinforcement interface above and below the reinforcement as the reinforcement is pulled out of the soil. This 'pullout' mode involves displacement of the reinforcement relative to the soil on both sides of the reinforcement. This is not often a problem for typical sheets of reinforcing material. However, it may occur if the reinforcement is in the form of strips or grids, or if insufficient room is available to anchor very high strength fabrics.

(b) If the shear strength of the soil reinforcement is less than the shear strength of the soil alone, then failure may occur by sliding of the soil along the upper surface of the reinforcement, as the upper soil mass moves relative to both the reinforcement and the underlying soil. This rarely occurs.

(c) The soil below the reinforcement (usually the soft foundation) may be squeezed out from beneath the lowest reinforcement layer (and the entire reinforced embankment). In this case, the soft foundation soil may move relative to the reinforcement and the overlying soil. This commonly occurs in reinforced embankment analysis.

If the reinforcement is in the form of a sheet, completely separating the soil above and below the reinforcement, then the interface resistance can be readily determined by direct shear tests. In this case, provision for slip at the interface is the same irrespective of the mechanism of failure (that is, direct shear or pullout). However, if the reinforcement takes the form of a geogrid, with openings which are large compared to the grain size of the soil, or if the reinforcement consists of separate reinforcing strips, then special care is required to correctly model the failure mechanism. For these materials, the interface shear resistance in direct shear (e.g. if there is sliding of the soil along the upper surface of the reinforcement) may be substantially higher than the interface resistance in pullout. In modelling these materials, it is necessary for the formulation of the interface element to be such that it can detect whether it is in a direct shear or pullout mode and to then select the appropriate interface parameters to model this mode of
shearing. Thus the behaviour of the interface element on one side of the reinforcement is related to the behaviour of the interface element on the other side (since the mode of shearing can only be assessed by consideration of the direction of shear on either side of the reinforcement).

For planar reinforcement, independent movement of the soil may occur above and below the reinforcement following either a direct shear or pullout failure. For strip reinforcement, independent movement of the soil above and below the plane of reinforcement can only occur during a direct shear mode of failure. Pullout of strips is really a three-dimensional phenomenon, in which the strips move relative to the soil around them but the soil between strips remains continuous. As noted by Naylor and Richards, the common approach of using a conventional joint element (or nodal compatibility element) implicitly treats the strips as an equivalent two-dimensional sheet and will cause serious error since it interrupts the transfer of shear stress through the soil.

Since pullout of strips does represent a truly three-dimensional situation, it can only be approximately modelled in a two-dimensional analysis. A number of different approaches can be adopted. For example, Naylor and Richards proposed a composite formulation which ensured continuity of shear stress in the soil after pullout by introducing a 'conceptual shear zone'. An alternative approach implemented by the present authors in their formulation involves an interface element which has a node above the reinforcement, a node on the reinforcement and a node below the reinforcement. Prior to slip, normal and tangential compatibility between the soil and reinforcement is enforced by means of very stiff springs. The normal and shear stresses 'above' and 'below' the reinforcement are automatically monitored. If a pullout mode of failure occurs (as inferred by the direction of shear above and below the reinforcement together with a Mohr–Coulomb failure criterion), then the computer program automatically enforces compatibility between the soil nodes 'above' and 'below' the reinforcement (thereby maintaining continuous transfer of shear stress in the soil) while allowing slip between the reinforcement node and the two soil nodes. The normal force between these nodes is used to assess the normal forces acting on the strip; the corresponding shear resistance (based on a Mohr–Coulomb failure criterion) between the strip and soil is applied to both the upper and lower soil node, and as an equilibrating force to the node on the soil strip. Since the strip covers only a small area of the soil, the Mohr–Coulomb parameters must be adjusted to take account of the actual surface area, per unit width of the embankment, which is in contact with the soil.

For the remainder of this paper, attention will be restricted to sheets of geosynthetic reinforcement. Rowe and Mylleville discuss strip reinforcement.
3 NUMERICAL PROCEDURE

The previous section discussed factors associated with the selection of the finite elements and constitutive model to be used. The validity of finite element results will, of course, be dependent on the use of a suitable finite element model. However, the choices of load step size and finite element mesh together with the numerical procedure adopted for ensuring that the failure criteria, the flow rule and total equilibrium are all satisfied at the end of each load increment are equally important.

The construction sequence adopted in the finite element simulation should follow, as nearly as practicable, that which would be anticipated in practice. Typically (but not necessarily—e.g. in the case of dredging) this will involve construction of the embankment in a number of horizontal layers or lifts. Each of these layers will be simulated by a row of finite elements where construction of the layer involves ‘turning on gravity’ (i.e. increasing the unit weight of the layer from zero up to the ‘design value’) in a number of increments. For non-linear problems involving both plasticity and stress-dependent fill stiffness, the choice of load step size should be such that any further reduction in load step size will not significantly affect the results of the analysis. This can be established by repeating an analysis for a number of different load step sizes. For the analyses to be discussed in the following section, the construction simulation involved up to 16 lifts (layers) of fill and a total of up to 250 load steps (increments) being placed.

The finite element mesh should be selected such that it is sufficiently refined (i.e. has enough degrees of freedom) in the critical zone where collapse occurs to ensure that the collapse height and mechanism are determined to sufficient accuracy. A preliminary indication of how this critical zone may change due to reinforcement can be obtained by looking at critical circles from limit equilibrium analyses for the unreinforced case, and by examining the plasticity solutions for a rigid footing (to be discussed in a following section).

The validity of the finite element mesh and procedure can be assumed by comparing the finite element collapse height for an unreinforced embankment with that from conventional limit equilibrium analyses and by comparing the collapse height for a very heavily reinforced embankment with that estimated from plasticity solutions for rigid footings.

4 EFFECT OF REINFORCEMENT ON FAILURE MECHANISM

As intimated in the preceding section, the inclusion of high modulus reinforcement in an embankment can significantly change the failure
mechanism. For example, Figs 1 and 2 show the velocity field at collapse obtained by Rowe and Soderman\textsuperscript{15} from a finite element analysis of an unreinforced ($J = 0$) and a heavily reinforced (reinforcement modulus $J = 4000$ kN/m) embankment\textsuperscript{1} having a crest width of 30 m (2:1 side slopes) and resting on a soil with an undrained shear strength which increases linearly from a surface value $c^*_s$ of 7.69 kPa at a rate $\rho^*_s = 1.54$ kPa/m. In these 'velocity fields' the arrows indicate the direction and relative magnitude of the soil movements at the onset of collapse.

In Fig. 1, the velocity field indicates that at a collapse height of 3 m, the failure mechanism for the unreinforced embankment begins a few metres from the shoulder and extends to a depth of about 2 m. For comparison purposes, the critical circle from a simplified Bishop limit equilibrium analysis is also shown and it can be seen that the two methods of analysis give a very similar failure mechanism and collapse height.

The collapse height obtained for the heavily reinforced embankment ($J = 4000$ kN/m) is approximately twice that for the unreinforced embankment. The reason for this substantial increase in collapse height is evident from a comparison of Figs 1 and 2, which shows that the reinforcement forces the collapse mechanism down into the stronger soil at depth. In fact, increasing the modulus from 0 to 4000 kN/m moves the edge of the mechanism from near the shoulder to near the centreline of the embankment and forces it from a depth of about 2 m to a depth of between 8.5 and 9 m.

The preceding example is one in which the finite element results demonstrate that high modulus reinforcement can substantially improve the stability of an embankment and also, by inspection of the change in failure mechanism, provides an intuitive feel for why this improved performance was realized. However, the finite element solution can be equally useful for indicating situations where even high modulus reinforcement may give rise to very little improvement in stability. To illustrate this, Figs 3 and 4 show the velocity fields at collapse obtained by Rowe and Soderman\textsuperscript{13} for an unreinforced embankment constructed on a deposit of clay having a uniform strength with depth. The primary difference between the two sets of results is the thickness of the deposit, which corresponds to a depth to crest width ratio ($D/B$) of 0.33 and 0.55 for Figs 3 and 4 respectively.

An examination of the displacement components at the interface between the fill and foundation in Figs 3 and 4 reveals that for $D/B = 0.33$, the component of horizontal displacement is substantially greater than for

\textsuperscript{1}For the analyses discussed in this paper, the fill properties were assumed to be given by friction angle $\phi = 32^\circ$, dilatancy $\psi = 0^\circ$, unit weight $\gamma = 20$ kN/m$^3$, Poisson's ratio $\nu = 0.35$ and dimensionless stiffness parameters $K = 100$ and $m = 0.5$.
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Fig. 1. Velocity field at collapse for the case $c_{\omega}^* = 7.69$ kPa, $\rho_c^* = 1.54$ kPa/m and $J = 0$ (unreinforced) (after Ref. 15).

Fig. 2. Velocity field at collapse for the case of $c_{\omega}^* = 7.69$ kPa, $\rho_c^* = 1.54$ kPa/m and $J = 4000$ kN/m (after Ref. 15).

Fig. 3. Velocity field after collapse, $D/B = 0.33$ (after Ref. 13).
If a strong reinforcing geotextile was to be inserted at the interface between the fill and foundation, then for $D/B = 0.33$ one would expect that the reinforcement would resist these lateral movements thereby mobilizing forces in the reinforcement, changing the failure mechanism to give a more rigid downward movement of the embankment and, consequently, increasing the collapse height. However, for $D/B = 0.55$, there is very little lateral movement at the interface and the inclusion of a reinforcing geotextile would not be expected to significantly change the failure mechanism or the collapse height for this case.

The results presented in this section show that the improvement in stability which can be achieved by using a given high strength geosynthetic reinforcement will depend on the undrained strength profile and relative depth of the deposit, and as a consequence, it may be very unwise to extrapolate potential benefit of reinforcement obtained in field trials on one site to another site.

5 ESTIMATING THE MAXIMUM IMPROVEMENT IN COLLAPSE HEIGHT THAT CAN BE OBTAINED USING REINFORCEMENT

A number of investigators\textsuperscript{35,36} have developed bearing capacity factors for rigid footings. These solutions have considered the effect of increasing undrained strength with depth as well as the effect of the relative thickness of the soil deposit. The bearing capacity factors determined by them have been synthesized and plotted in Fig. 5 in terms of the dimensionless quantity $\rho_c b/c_u$, where $\rho_c$ is the rate of change in $c_u$, $b$ is the effective width of the footing, and $c_u$ the undrained strength directly beneath the footing (see Fig. 5). Since a reinforced embankment can never be reinforced beyond the
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Fig. 5. Bearing capacity factor for non-homogeneous soil (synthesized from results by Davis and Booker,\textsuperscript{35} and Matar and Salencon.\textsuperscript{36}

point of being rigid, these solutions place a limit on the improvement in stability which can be achieved using high strength reinforcement. In conjunction with a finite element analysis, these solutions may serve two purposes.

Firstly, prior to the analysis, they can be used to estimate the maximum height of embankment which can be achieved and this information will influence the design of the finite element mesh. Secondly, after an analysis, these solutions can be used as a check on the collapse height obtained from the finite element analysis for a highly reinforced embankment.

Since the plasticity solutions are for a rigid footing of width $b$ and since the embankment will generally have a trapezoidal shape, an approximation must be made to determine the equivalent width of the embankment. From plasticity considerations, the pressure at the edge of a rigid footing is $(2 + \pi)c_{\text{w}}$. It is assumed here that the effective width of the footing $b$ will extend between the points on either side of the embankment where the applied pressure $\gamma h$ is equal to $(2 + \pi)c_{\text{w}}$. Thus, the thickness $h$, where the applied pressure is $(2 + \pi)c_{\text{w}}$, is

$$h = \frac{(2 + \pi)c_{\text{w}}}{\gamma}$$

and hence (from Fig. 6)

$$b = B + 2n(H - h)$$
Fig. 6. Definition of variables used to estimate collapse height for a perfectly reinforced embankment.

![Diagram of embankment and rigid footing]

Fig. 7. Effect of non-homogeneity on depth of the failure zone beneath a rough rigid footing (modified from Ref. 36).

where \( B \) is the crest width, \( H \) is the embankment height and \( n \) is the cotangent of the slope angle.

The bearing capacity \( q_u \) of the rigid footing of width \( b \) is given by

\[
q_u = N_c c_{uo} + q_s
\]  

(4)

where \( q_s \) is a uniform surcharge pressure applied to the soil surface outside of the footing width and \( N_c \) is the bearing capacity factor obtained from Fig. 5. Inspection of Fig. 6 shows that the triangular edge of the embankment is providing a surcharge that would increase stability. What is required is an estimate of \( q_s \) in terms of the pressure applied by this triangular distribution.

Figure 7 shows the depth \( d \) to which the failure mechanism is expected to extend based on Matar and Salencon.\(^{36}\) From an inspection of typical characteristic fields, it is found that the lateral extent of the plastic region
involved in the collapse of a rigid footing extends a distance \( x \) from the footing where \( x \) is approximately equal to the minimum of \( d \) as determined from Fig. 7, and the actual thickness of the deposit \( D \), i.e.

\[
x = \min(d, D)
\]

Thus distributing the applied pressure due to the triangular distribution over a distance \( x \) gives

\[
q_s = n\gamma h^2/2x \quad \text{for } x > nh
\]

and

\[
q_s = (2nh - x)\gamma h/2nh \quad \text{for } x \leq nh
\]

This value may then be compared with the average applied pressure \( q_a \) due to the embankment over the width \( b \), viz.

\[
q_a = \gamma [BH + n(H^2 - h^2)]/b
\]

At collapse, the ratio \( q_u/q_s \) should be equal to unity. Thus, to determine the collapse height, \( H_c \), for a given crest width, side slope and strength profile, it is necessary to adopt a procedure in which \( H \) is assumed, \( q_u \) and \( q_s \) are calculated, and the ratio of \( q_u/q_s \) determined. If \( q_u/q_s \) is greater than unity, then \( H \) should be increased (if less than unity, then \( H \) is decreased) until the critical height \( H = H_c \) is achieved wherein \( q_u/q_s = 1 \). This is the procedure to be adopted initially to determine the maximum height of embankment that may need to be modelled.

For the purposes of estimating the maximum possible factor of safety (defined here as \( FS = q_u/q_s \)) for a given embankment geometry and soil profile, or for checking the reasonableness of a finite element collapse load, \( q_u \) and \( q_s \) can be determined directly from eqns (2)–(7), and hence the ratio \( q_u/q_s \) can be determined. In the case where the finite element analysis indicates collapse, the ratio of \( q_u/q_s \) calculated for the embankment geometry at collapse should be approximately equal to unity for a perfectly reinforced embankment (i.e. provided that failure is not controlled by failure or yield of the reinforcement).

To illustrate the application of this checking technique, consider the highly reinforced embankment whose velocity field at collapse is shown in Fig. 2. Figure 8 shows the vertical displacement at a point \( a \) beneath the shoulder of the embankment, as the height of the embankment is increased (i.e., as \( \gamma H \) increases, where \( H \) is the total thickness of the fill above point \( a \) at any time). Results are given corresponding to fabric modulus values of 1000, 2000 and 4000 kN/m.
In each case, the deformations of the embankment became indeterminate and collapse occurred at an applied pressure of approximately 120 kPa. The collapse pressure was independent of the geotextile modulus and was in fact controlled by shear failure at the interface between the geotextile and the underlying clay foundation. Thus, for the case where $B = 30 \text{ m}$, $H_c = 6 \text{ m}$, $\gamma = 20 \text{ kN/m}^3$, $c_{u0} = 7.69 \text{ kPa}$, $\rho_c = 1.54 \text{ kPa/m}$ and $n = 2$, from eqns (2) and (3) it is found that $h = 1.98 \text{ m}$ and $b = 46 \text{ m}$. This gives $\rho_c/b/c_{u0}$ equal to 9.2 and hence from Fig. 5 $N_c = 12.3$. For $\rho_c/b/c_{u0} = 9.2$, Fig. 7 gives $x = 0.2$, $b = 9.2 \text{ m}$, and hence from eqn (6a), $q_s = 8.5 \text{ kPa}$, and so from eqn (4) the ultimate bearing capacity is 103 kPa. This may be compared with the average applied pressure in the finite element analysis, which from eqn (7) is 106 kPa. Thus the calculated applied pressure exceeds the ultimate bearing capacity from plasticity theory by about 3%. This level of ‘error’ associated with the numerical solution is considered to be acceptable for typical finite element analyses.

6 USE OF FINITE ELEMENT ANALYSIS TO DETERMINE FAILURE AND COLLAPSE HEIGHTS

As demonstrated in the previous section, classical plasticity solutions can be used to determine the maximum improvement in stability which can be achieved by the use of reinforcement. Furthermore, the results shown in Fig. 8 suggest that for a perfectly reinforced embankment, the collapse load
is independent of the modulus of the geosynthetic. However, this collapse load does not take account of possible failure of the reinforcement or pullout of the reinforcement, nor does it consider the magnitude of the deformations that may develop prior to 'collapse'. This then raises the question as to what is the collapse height for a given situation and whether there is a height less than the collapse height at which the embankment may be deemed to have failed, even though, in the strict sense of plasticity theory, it has not collapsed. To answer this question, it is useful to define the term 'contiguous plasticity' and to look at the sequence of events as a reinforced embankment is constructed up to collapse as determined from a finite element analysis.

The term 'contiguous plasticity' relates to the situation where there is general plastic failure within the soil in the region of a potential collapse mechanism (i.e. the shear strength of the soil is mobilized along the potential collapse mechanism). For an unreinforced embankment, collapse and failure coincide with the development of contiguous plasticity and a corresponding collapse mechanism (see Fig. 1). However, for a reinforced embankment, the development of contiguous plasticity is only the first step towards failure and collapse. The height at which contiguous plasticity is developed is dependent upon the geotextile modulus as indicated in Fig. 8, but the effect is not large. The effect of the geotextile modulus is really only appreciable after the development of contiguous plasticity. Thus although the collapse height is the same for the three cases examined in Fig. 8, the deformations prior to collapse differ substantially.

The importance of considering deformations in any assessment of failure can be demonstrated by replotting the results given in Fig. 8 in the form of net fill height above original ground level (i.e. the thickness $H$ minus the vertical deflection for that fill thickness) against the fill thickness $H$ as shown in Fig. 9. For the unreinforced embankment, the maximum net fill height is about 3 m and occurs at the onset of contiguous plasticity. Notice that this also corresponds to the collapse height and in fact the net fill height at the onset of collapse is only slightly less than the fill thickness. For the reinforced embankments, it can be seen that the maximum net fill height does depend on the geotextile modulus. Any attempt to place additional fill after attaining this maximum height will result in the loss of net height. This corresponds to a controlled failure. In order to support the stresses imposed by the additional fill, large deformations of the geotextile and the soil must develop. This can continue in a controlled manner until collapse eventually occurs. However, from a practical standpoint, the fill thickness of the embankment at failure should be considered to correspond to the fill thickness at the time the maximum net fill height is obtained and not the fill thickness required to achieve 'true' (i.e. uncontrolled) collapse.
From the foregoing it is evident that the failure height of an embankment on a given soil profile is directly related to the modulus of the geotextile. As might be expected, the maximum force mobilized in the geotextile at failure increases with increasing geotextile modulus. However, because the deformation pattern also changes with increasing geotextile modulus, these forces do not correspond to a unique strain for soil profiles where the strength increases with depth.

Since the failure height of an embankment, as defined above, is related to deformations, one would expect that the modulus of the soil would have some influence on the calculated failure height for a given level of reinforcement.

The ratio of undrained Young's modulus to undrained shear strength \((E_u/c_u)\) for many soft clays lies in the range \(125 \leq E_u/c_u \leq 500\). The results presented in Figs 1, 2, 8 and 9 were obtained for \(E_u/c_u = 125\). Figure 10 shows the effect of varying \(E_u/c_u\) from 125 to 500. This fourfold increase in soil modulus reduced the embankment thickness required for contiguous plasticity by about 6% and reduced the maximum geotextile strain at contiguous plasticity from 2.5% to 1.0%. The embankment thickness required to cause failure and the maximum geotextile strain at failure \((\sim 7.5\%)\) were to all practical purposes the same, although the maximum net height was slightly smaller for the lower value of \(E_u/c_u\). It may be concluded that the failure pressures and maximum geotextile strains at failure obtained for
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$E_u/c_u = 125$ are also reasonable for values of $E_u/c_u$ in the range $125 < E_u/c_u < 500$. The explanation for this lies in the fact that the strains which control failure are predominantly plastic strains and, since changing the modulus really only changes the elastic strains (and these are small), there is no significant effect.

7 DEVELOPMENT OF STRAIN IN THE REINFORCEMENT AND FACTORS OF SAFETY

Rowe and Soderman have compared the calculated and observed performance of both a reinforced and an unreinforced test embankment constructed on a 3.3 m thick layer of very soft organic clay ($c_u = 8$ kPa) at Almere, The Netherlands. The construction procedure for the reinforced embankment involved (a) placing a high modulus geotextile ($J = 2000$ kN/m) directly on the clay foundation, (b) excavation of a ditch beside the proposed embankment and placing the soil on top of the geotextile near the edge to form a retaining bank (see insert to Fig. 11), and (c) placing hydraulic fill until failure occurred. The strains in the geotextile were monitored during construction. Figure 11 shows the observed and calculated increase in strain at point A (see insert) after the commencement of hydraulic filling. As can be seen, very little strain is developed during initial
stages of loading (until the fill height reaches about 1 m). There is then a gradual increase in strain with increasing fill height between 1 and 2 m, and a rapid increase in strain for fill heights in excess of 2 m. Collapse occurred when the fill height was 2.75 m. The fill height depends upon the position where it was measured (see insert to Fig. 11). The minimum and maximum heights at collapse were 2.75 m and 3 m respectively. The lower height is used herein as the reference height.

The analysis indicates that for fill heights less than 1 m, the clay is largely elastic and the lateral deformations are small. As the fill is increased from 1 m to 2 m, there is extensive growth of the plastic region within the clay, giving rise to an increase in lateral and vertical deformations which is, in turn, reflected by increased strain in the fabric. At a given height, the geotextile reduces plasticity within the soil. For example, in the un-
reinforced case, the analysis predicted failure at a fill height of 1.8 m. At the same height in the reinforced embankment, the displacements are smaller and the plastic region was not contiguous. A contiguous plastic region in the soil was calculated to occur at a fill height of 2.05 m. This is approximately 15% higher than the corresponding height for the unreinforced embankment. The development of a contiguous plastic region represents the first stage of collapse for the embankment; the embankment is now completely dependent upon the geotextile for the support of additional fill. Further fill was added until, at a height of approximately 2.7 m, the analysis indicated that the shear strength of the soil–geotextile interface was reached and pullout occurred beneath the clay retaining bank. This was followed by collapse of the embankment. Since the geotextile force at point A was controlled by the clay–geotextile interface strength and pullout to the right of A (see insert to Fig. 11), there could not be any increase in strain at this point.

Three observations can be drawn from this case. Firstly, the use of a high modulus geotextile substantially increased the collapse height (from 1.75 m unreinforced to 2.75 m reinforced). Secondly, the geotextile did not play a significant role, and hence did not experience significant increases in strain, until a large area of plastic failure had developed in the underlying foundation. Finally, the agreement between the calculated and observed behaviour was very encouraging.

The observed rapid development of strain once contiguous plasticity is reached is to be expected. However, since the height at which contiguous plasticity occurs is only slightly increased by the inclusion of a high modulus fabric (in the Almere case from 1.8 m unreinforced to 2.05 m reinforced), this raises the question as to what magnitude of strains may be developed at failure and what magnitude of strains would be developed under working conditions.

In the context of this paper, the 'factor of safety' is defined as the divisor necessary to reduce the soil strength to such a point that failure would just occur. Thus, for example, if the expected undrained shear strength parameters for a foundation soil were $c_{uo} = 10$ kPa and $\rho_c = 2$ kPa/m, then the allowable height for an embankment which has a 'factor of safety' of 1.3 would be the height at which failure occurs in an analysis conducted for the factored parameters $c^*_{uo} = 10/1.3 = 7.69$ kPa and $\rho^*_c = 2/1.3 = 1.54$ kPa/m. To illustrate the implications of this, Figs 12 and 13 show the development of the maximum strain in the geotextile with increasing embankment height $H$, as calculated for the factored parameters ($c^*_{uo}$, $\rho^*_c$) and the expected parameters ($c_{uo}$, $\rho_c$) respectively.

In both cases, the trends indicated by the analysis are similar to that observed at the Almere test embankment. Thus the geotextile strain in-
Fig. 12. Applied pressure versus maximum geotextile strain for the case of $c_{w0}^* = 7.69$ kPa, $\rho_c^* = 1.54$ kPa/m ($J = 4000$ kN/m, $B = 30$ m, $n = 2$, $D = 15$ m, $\gamma = 20$ kN/m$^3$).

Fig. 13. Applied pressure versus maximum geotextile strain for the case of $c_{w0} = 10$ kPa, $\rho_c = 2$ kPa/m ($J = 4000$ kN/m, $B = 30$ m, $n = 2$, $D = 15$ m, $\gamma = 20$ kN/m$^3$).
creases slowly until the height corresponding to contiguous plasticity is reached. The embankment is then completely dependent on the geotextile for stability and the strain increases rapidly until failure occurs. The primary difference between these analyses and the Almere analysis was that here the fill was all assumed to be granular and hence failure was controlled by the foundation and geotextile modulus and not by pullout.

Using factored parameters \( (c_{\text{uo}}^* = 7.69 \text{ kPa}, \rho_c^* = 1.54 \text{ kPa/m}) \), failure occurred at a pressure \( \gamma H \) of 118 kPa (i.e. \( H = 6 \text{ m} \)) and the corresponding geotextile strain at failure was 7.5%. Thus if one adopts a factor of safety of 1.3, then this height of about 6 m would be the allowable height for an embankment with design parameters \( c_{\text{uo}} = 10 \text{ kPa}, \rho_c = 2 \text{ kPa/m} \). It should also be noted that if the permissible strain in the fabric was 10%, then this would also correspond to a factor of safety of approximately 1.3 against reaching the permissible strain (i.e. \( FS = 10/7.5 = 1.33 \)).

Figure 13 shows the results of an analysis performed using the expected parameters \( (c_{\text{uo}}, \rho_c) \). It can be seen that under expected working conditions (i.e. at an allowable pressure of 118 kPa based on the analyses for the factored parameters), the maximum geotextile strain is only about 3%. This is considerably less than the maximum geotextile strain of 7.5% which occurred at the pressure of 118 kPa under failure conditions for the factored strength parameters. This situation arises because most of the strain is developed after contiguous plasticity is reached. Using the factored parameters (see Fig. 12), contiguous plasticity occurred at an embankment height of 4 m, while failure did not occur until a height of 6 m (i.e. 50% higher than the contiguous plasticity height) was reached, with the geotextile strains increasing from 1.5% to 7.0% as the embankment height increased from 4 m to 6 m. With the expected parameters, the height at which contiguous plasticity would occur is 5.5 m, which is only about 10% below the design height of 6 m. Under these working conditions, the 'factor of safety' with respect to an assumed permissible stress of 10% is 3.3 as opposed to 1.3 based on factored conditions.

This example illustrates that the strains in the geotextile are very sensitive to the height of the embankment relative to the height at which contiguous plasticity would occur. Thus, for a given design height, the geotextile strains will be very sensitive to the magnitude of the actual shear strength of the soil.

The failure pressures obtained for these two cases can be used to contrast two possible definitions of the factor of safety. With the definition of the factor of safety adopted above (i.e. based on factored shear strength parameters), the allowable pressure is 118 kPa for the case of nominal strengths \( c_{\text{uo}} = 10 \text{ kPa} \) and \( \rho_c = 2 \text{ kPa/m} \), a geotextile of modulus 4000 kN/m and an overall factor of safety of 1.3. Alternatively, the factor of safety could be defined as the ratio of the failure pressure obtained using nominal
strengths to the allowable pressure. This definition yields a factor of safety of 1.42 at the allowable pressure of 118 kPa suggested previously. This calculation indicates the latter definition of the safety factor is less conservative than the definition based on factored strength. Since the major uncertainty in most embankment designs is the shear strength of the foundation, the authors would recommend the use of factored strength to determine allowable design heights.

It is also noted that the magnitude of strain in the geosynthetic reinforcement at failure will depend on the geometry of the embankment, the shear strength characteristics of the foundation and the modulus of the reinforcement. As a consequence, there is no simple 'limiting geotextile strain' that can be safely or economically applied to all situations (e.g. see Ref. 15).

CONCLUSION

The finite element technique is a very powerful tool which can be used in the analysis of reinforced embankment behaviour. However, the applicability of the results for any given situation will depend on the details of the specific finite elements and constitutive model which are used, as well as on the care with which the numerical analysis is conducted. Thus finite element schemes should be validated against available benchmark solution (as illustrated in this paper) and, wherever possible, against observed field performance.

Results from a number of finite element analyses performed by the authors have been discussed. The finite element results were shown to be particularly useful for identifying the mechanisms of failure and also for indicating why a geosynthetic reinforcing material may substantially improve stability for a certain foundation strength profile, whereas for different foundation strength profiles the same reinforcement may give rise to negligible improvement in embankment stability.

The use of plasticity solutions developed for a rigid footing, for estimating the maximum effect of reinforcement was illustrated in the paper. The results of finite element analyses were used to demonstrate that although the collapse load calculated from plasticity theory can be attained for very highly reinforced embankments, in many situations failure will occur at embankment heights well below the collapse height. It is then demonstrated that the failure height for a reinforced embankment is related to the modulus of the reinforcement. As might be expected, the maximum force mobilized in the geotextile at failure increases with increasing geotextile modulus. However, because the deformation pattern in the soil also changes with increasing modulus (due to different levels of plasticity in the soil),
these forces do not correspond to a unique strain that could be generalized as a standard limit on strain to be used in limit equilibrium analyses.

It is shown that although failure has been defined in terms of a displacement criterion (specifically, failure is deemed to have occurred when the addition of fill would not give rise to any additional increase in embankment height above original ground level), it is shown that the failure height is not significantly influenced by the modulus of the soil over a typical range of modulus values expected in most field situations. It is concluded that this situation arises because failure is controlled by plastic strains in the soil (not elastic strains) and the modulus of the reinforcement.

The development of strain in the geotextile was examined, and it was demonstrated both from field evidence and theoretical analysis that the reinforcement plays a relatively small role at low load levels since the soil is essentially elastic. Significant strain in the geotextile begins to develop with increasing plasticity and in fact most of the strain is developed after a contiguous plastic region is developed in the soil, since beyond this point the reinforcement is all that prevents collapse from occurring. As a consequence, the strains developed in reinforcement for a given embankment height will largely depend on the height of the embankment relative to the height at which contiguous plasticity occurs, and hence will be sensitive to the magnitude and distribution of the actual shear strength in a soil deposit.

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