2D modelling of clogging in landfill leachate collection systems

A.J. Cooke and R. Kerry Rowe

Abstract: A 2D model for predicting clogging of a landfill leachate collection system and subsequent leachate surface position (mounding) is described. A transient finite element fluid flow model is combined with a reactive, multiple-species finite element transport model. The transport model considers biological growth and biodegradation, precipitation, and particle attachment and detachment. It uses a geometrical relationship to establish porosity from the computed thickness of the accumulated clog matter and a relationship between the porosity and hydraulic conductivity of elements in the system. The model represents the flow path within the drainage layer in profile. An iterative method is used to solve for the new hydraulic heads, surface and internal nodal positions, and redistributed clog properties (clog quantity, porosity, hydraulic conductivity) for each element and for each time step. The porosity (and consequently hydraulic conductivity) of the media can therefore change spatially and temporally. The mesh is regenerated automatically each time step (including the addition or subtraction of nodes) taking into account allowable element aspect ratios, the interfaces between differing hydro-stratigraphic layers, and static point sources and openings. An integrated alternate solution for very thin mounds is included. The application of the model is demonstrated using a hypothetical field case.

Key words: clogging, landfills, leachate collection systems, modelling, biofilms, mineral precipitation.

Introduction

The leakage through landfill barrier systems has received considerable interest with most of the focus having been on the performance of the composite liner (Rowe and Brachman 2004; Rowe 2005; Barroso et al. 2006; Dickinson and Brachman 2006; Touze–Foltz et al. 2006; Rowe et al. 2007; Saidi et al. 2008). However, the key driver for leakage through these liners is the leachate head on the liner. Thus the primary function of the leachate collection system (LCS) in modern municipal solid waste landfill is to control the leachate head on the underlying liner, thereby minimizing advective flow (leakage) through the liner to the environment. This is commonly achieved using a continuous high permeability drainage layer (e.g., sand, gravel) in conjunction with regularly spaced pipes (e.g., Fig. 1) that lead to sumps from which the leachate is removed for treatment. The leachate mound in the LCS is a function of the rate of infiltration, pipe spacing, bottom slope, and the hydraulic conductivity of the drainage layer.

In some countries the maximum design head is stipulated as a design criterion (typically it is not to exceed 0.3 m except for very short periods). For some time it was common design practice to use “Moore’s Equations” (Moore 1980,
1983; USEPA 1989) to estimate the maximum depth of leachate. However, with time, the validity of the Moore equations was questioned, and since no derivation was published, it has became more common to use Giroud’s equation (J.P. Giroud, *Height of leachate in a drainage layer*, unpublished project notes 1985) and the subsequent improved version by Giroud et al. (1992), or the equations of McEnroe (1989, 1993). These solutions are analytical and are based on a number of simplifying assumptions, most significantly: (i) flow is essentially horizontal (thus permitting use of the Dupuit approximation), (ii) hydraulic conductivity is constant and homogeneous, and (iii) infiltration is constant and uniform. Of these solutions, only McEnroe (1989) predicted the entire steady-state profile of the phreatic surface.

More rigorous methods (intended for unconfined flow in general) often employ finite element routines wherein the mound surface is a boundary whose position is initially unknown and must be obtained iteratively. This type of analysis was pioneered by Taylor and Brown (1967) for steady-state conditions and by Neuman and Witherspoon (1971) for nonsteady flow through homogeneous soils. More recently, Crowe et al. (1999) presented a variable mesh technique that allowed for the surface to rise and fall within heterogeneous soils.

Of the factors responsible for leachate depth in the LCS, it is the hydraulic conductivity that is the hardest to control or predict after landfill operation has commenced. Field observations (see Bass 1986; Brune et al. 1991; McBean et al. 1993; Rowe et al. 1995; Rowe 1998; Craven et al. 1999; Fleming et al. 1999; Maliva et al. 2000; Rowe et al. 2004) have concluded that the transmission of leachate through the drainage layer leads to the build-up of clog material and a reduction in hydraulic conductivity. Prediction of clog build-up (clogging) within the drainage layer and the subsequent profile of the mound, as they change temporally and spatially, would aid in the design of new landfill LCSs. At the time of writing the authors were unaware of any published model with this objective.

As a first step towards modelling the clogging of the LCS, a numerical model (called BioClog) was developed to apply to column experiments in which glass beads were permeated with synthetic leachate (Cooke et al. 1999, 2001). This one-dimensional (1D) flow model was updated significantly with additional mechanisms to better predict clogging caused by real leachate which contained significant suspended solids (Cooke et al. 2005a). Using the model, clogging of experimental columns packed with glass beads and permeated with synthetic and real leachate were successfully modelled by VanGulck et al. (2003), and columns packed with gravel and permeated with real leachate were successfully modelled by Cooke et al. (2005b). The objective of this paper is to describe the incorporation of the previously developed and tested clogging processes into a 2D system for the purposes of predicting the change in the leachate mound in the LCS with time and hence to allow estimates to be made of the likely performance of different LCS designs.

The following sections describe the problem domains that can be represented by the BioClog model and the technique used to model flow in a variable mesh system. Subsequently, the methods by which species transport, clogging, and species fate are modelled is discussed. This is followed by an elaboration on the issues relevant to the problem. Lastly, the application of the model is illustrated for several hypothetical field cases.

**Problem definition**

Consider a representative 2D cross section (e.g., the “modelled zone” in Fig. 1) through a LCS with a perforated leachate collection pipe perpendicular to the cross section being the means by which leachate is removed from the cross section. This will be modelled using a 2D finite element formulation. The initial and subsequent versions of the 2D mesh are produced by the model based on the calculated location of the phreatic surface, the location of the bottom of the mesh, and the positions of layer interfaces and boundary conditions that affect nodal placement (described later). The inclusion of layers allows the model to consider a sand protection layer beneath a gravel drainage layer and (or) a sand filter above a gravel drainage layer as well as the waste. The current implementation uses three node linear triangle elements although the concepts could be readily used in conjunction with higher order elements.

**Typical flow boundary conditions**

The percolation of leachate out of the waste above the drainage layer was modelled by specified (uniform) vertical vertical
infiltration into the upper row of elements representing the top of the leachate mound (i.e., at the upper boundary). The base below the drainage layer was assigned “no flow” conditions. To allow the modelling of field LCS as shown by the “modelled zone” in Fig. 1, symmetry implies a “no flow” boundary at the left boundary. The presence of a perforated leachate collection pipe at the right boundary was modelled as an “open” zone with the zones above and below the pipe being “no flow” zones due to symmetry (Figs. 1 and 2a). Within the pipe (open zone), hydrostatic conditions were assigned below the specified head corresponding to the assumed level of leachate flow in the pipe, and atmospheric conditions were specified above this level (this zone was a potential seepage face). For modelling of the left boundary in a 2D mesocosm test (Fleming and Rowe 2004; McIsaac and Rowe 2007), a static point source (Fig. 2b) or distributed specified fluid flux (Fig. 2c) can be used to simulate the lateral inflow of leachate into the system with “no flow” conditions above and below the lateral fluid source.

Typical solute transport boundary conditions

For solute transport, the model was programmed to include specified concentration (Dirichlet), specified flux (Neumann), specified total flux (Cauchy), and free exit (also called open) boundary conditions. The top boundary (the surface) may be assigned any one of the first three boundary conditions; although for the modelling reported herein a Cauchy condition was considered most realistic (see Padilla et al. 1997) and was therefore adopted. On the side boundaries all four of the boundary conditions may be assigned, although for the field case examined herein a Neumann condition (specifically, zero specified flux) was applied at the left “no flow” boundary and on the right boundary the open zone was given the free exit condition, with the remaining “no flow” zones given zero specified flux conditions.

Fluid flow modelling with variable mesh

Defining equations

The basis for the methods outlined here was first published by Neuman and Witherspoon (1971) for nonsteady modelling assuming constant hydraulic conductivity. It was necessary to modify the Neuman and Witherspoon approach for the present work to allow consideration of the nonsteady porosity and hydraulic conductivity (as discussed later). Unconfined, transient saturated fluid flow is modelled by solving

\[ \frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial h}{\partial z}\right) + q = 0 \]

where \( h \) is hydraulic head measured from the \( x \)-axis, \( k_x \) and \( k_z \) are components of saturated hydraulic conductivity in the \( x \) and \( z \) directions, respectively, and \( q \) is specified flow rate. For confined transient modelling, the term \( q \) would normally be a specific storage term (the rate of change in storage), which is a function of media and fluid compressibility and provides transient effects. For unconfined flow the effects of compressibility are negligible, and the transient behaviour is established in the boundary conditions of the free surface (see below).

The initial conditions for hydraulic head at each node, \( h \), and the elevation of the free surface nodes, \( Z_s \), at time \( t = 0 \) are given by

\[ [2] \quad h(x, z, 0) = h_0(x, z) \]
\[ [3] \quad Z_s(x, 0) = Z_{s,0}(x) \]

where \( h_0 \) and \( Z_{s,0} \) are the specified initial conditions.

The boundary conditions are defined as described below. Where hydrostatic conditions occur, specified head boundary conditions are applied using

\[ [4] \quad h(x, z, t) = H(x, z, t) \]

where \( H \) is equal to the specified elevation of the external fluid level. Where specified flux boundary conditions are used (including zero flux)

\[ [5] \quad k_x \frac{\partial h}{\partial x} n_x + k_z \frac{\partial h}{\partial z} n_z = -F(x, z, t) \]

where \( n_x \) and \( n_z \) are the \( x \) and \( z \) components, respectively, of the unit normal vector, and \( F \) is the specified fluid flux. At the seepage face (Fig. 2a) the head is equal to the elevation of the node, \( Z \)

\[ [6] \quad h(x, z, t) = Z \]

The free surface must satisfy two boundary conditions as both the heads and the node elevations are unknown; first, the atmospheric pressure condition, where the elevation of the free surface nodes must equal the hydraulic head, and second, the continuity condition whereupon the normal flux must correspond to the net effect of infiltration, \( I \), and specific yield, \( S_y \) (Neuman and Witherspoon 1971)

\[ [7] \quad Z_s(x, z, t) = h(x, z, Z_s, t) \]
\[ [8] \quad k_x \frac{\partial h}{\partial x} n_x + k_z \frac{\partial h}{\partial z} n_z = \left(I - S_y \frac{\partial Z}{\partial t}\right) n_z \]

The system of equations used to obtain values of hydraulic head at each node in the mesh, in matrix form, is

\[ [9] \quad \left(\omega [C] + (1 - \omega)[C]_t + \omega \Delta t[k]_{t+\Delta t}\right)\{h\}_{t+\Delta t} = \left(\omega [C] + (1 - \omega)[C]_t - (1 - \omega)\Delta t[k]_t\right)\{h\}_t + \Delta t\{(1 - \omega)\{F\}_t + \omega \{F\}_{t+\Delta t}\} \]

where \([C]\) is the specific yield matrix, \([k]\) is the global conductance matrix, \(\{h\}\) is the global flux vector, \(\omega \) is the relaxation factor, and \(\Delta t\) is the time step length. Note that \([C]\) is zero everywhere ex-

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Fig. 2. Typical flow boundary condition arrangements for representation of (a) field and laboratory conditions at a leachate collection pipe (with fluid flowing in the pipe in the specified head zone), and lateral inflow conditions using (b) a point source (as in the experiments by McIsaac and Rowe 2007), or (c) distributed specified fluid flux. The zones are defined by elevation, Z.

except at the free surface, where the boundary condition defined by eq. [8] is employed using

\[
[C]_e = \frac{S_y \Delta x_c}{6} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}
\]

and \(\Delta x_c\) is the horizontal distance between the free surface nodes of the element (Neuman and Witherspoon 1971). The global conductance matrix, \([k]\), is the same as is commonly employed for three node triangles (see Istok 1989). In this work, though, \([C]_e\) and \([k]\), are calculated using the nodal coordinates and porosity (which controls \(S_y\), \(k_y\), and \(k_z\)) at the previous time step, and \([C]_{n,\Delta t}\) and \([k]_{n,\Delta t}\) are calculated based on current properties. The specific yield, \(S_y\), is equal to the element porosity, \(n\), taking clogging into account and hence varies with time. The relaxation factor \(\omega\) controls the finite difference formulation, with \(\omega = 0\) being forward difference, \(\omega = 0.5\) central difference, and \(\omega = 1\) backward difference. The backward difference formulation was used for the modelling herein.

Solution method

The system of equations (eq. [9]) cannot be used to solve \(h_{i+\Delta t}\) directly because the nodal elevations at time \(t + \Delta t\) are unknown. Additionally, \(n\), \(S_y\), \(k_y\), and \(k_z\) are considered to be unknown at time \(t + \Delta t\) because these parameters change as the clog material is redistributed according to changes in element location. To solve for all of these parameters each time step, the model uses two nested iterative routines; an inner routine to solve for \(h_{i+\Delta t}\) and \(Z_s\) and an outer routine to solve for \(n\), \(S_y\), \(k_y\), and \(k_z\). Initial element properties \((n, S_y, k_y, k_z)\) are calculated based on the elevation of the nodes from the previous time step. The element properties are kept constant while the surface heads, \(h_{i+\Delta t}\), and elevations, \(Z_{n,\Delta t,i}\), are adjusted until convergence is achieved (satisfying eq. [7]). The new nodal elevations are then used to recalculate the element properties, and the convergence routine is reiterated. The recalculation of the element properties is repeated until the surface elevation before convergence, \(Z_{a,j+\Delta t,i=1}\), and after convergence, \(Z_{a,j+\Delta t,i=ic}\), differ by less than a specified tolerance. Full details of the approach are described by Cooke (2007). The following highlights some important considerations for the modelling of a system where clogging is resulting in a change in element properties with time and position throughout the analysis. Additional details regarding techniques for improving computational efficiency and accuracy are provided by Cooke (2007).

Element properties (i.e., \(k\), \(n\), or specific surface \(A_s\)) are controlled by the thickness of five films (discussed subsequently) associated with each element. When node elevations are changed, element position and area is also changed. To maintain mass conservation for clog matter, the film thicknesses are redistributed. This procedure is also used when nodes are added or subtracted (as discussed later).

The three step approach originally proposed by Neuman and Witherspoon (1971) to solve for the head \(h_{i+\Delta t}\) method was found to result in relatively poor convergence for situations such as those examined here where the element properties change with time. Thus the following alternative approach was adopted. The specified head boundary condition was initially adopted for the seepage face and specified flux boundary condition for the free surface. All of the fluid fluxes along the free surface are functions of the infiltration rate alone, except at nodes, which are both part of the seepage face and free surface, where an unknown contribution to the flux passes through the seepage face. The fluid flux at these nodes is calculated from head conditions of the previous iteration (or previous time step, if this is the first iteration of a time step). The values of \(h_{i+\Delta t}\) are computed assuming the free surface boundary condition is that of eq. [8], where the normal fluid flux is equated to the net infiltration and specific yield. Using this boundary condition on the free surface produces new values of \(h_{i+\Delta t}\) for all nodes including the free surface, \(h_{i+\Delta t}\). The solution is taken to have converged when the calculated head \(h_{i+\Delta t}\) and the estimated node elevation \(Z_{n,i+\Delta t,i}\) are suitably close for all free surface nodes (and thus the boundary conditions of eqs. [7] and [8] are satisfied). Surface node elevations are repositioned, and heads are recalculated iteratively until con-
The volume of clog matter in each element is quantified by the thicknesses of five separate films: one biofilm for each of the three active substrate degraders, the inert biofilm, and the inorganic solids film. The rate at which active biomass, inert biomass, and inorganic particles attach to the porous media is computed using one of two methods: (i) the Rajagopalan and Tien (1976) filtration model, or, (ii) the Reddi and Bonala (1997) network model. The calculated rate of attachment is used to compute the quantity of suspended matter removed from suspension (appearing in the reaction term for the species in the transport model) and the increase in thickness of the associated film (each film thickness is recalculated every time step).

The utilization of substrate (VFA), and consequent substrate flux into the active biofilms, is modelled assuming Monod kinetics and molecular diffusion using the method of Rittmann and McCarty (1981). Based on the substrate flux and density of the active film, the additional thickness of biofilm due to growth is calculated using the Rittmann and Brunner (1984) approach. The rate at which mass is detached from the films and is added to the suspended concentration is calculated based on both shearing (Rittman 1982) and growth rate (Peyton and Characklis 1993). For the active biofilms (and active biomass in suspension), the rate at which mass is lost to decay contributes to the accumulation of inert film (or suspended inert biomass). The inert biomass and inorganic solids films do not lose mass to decay.

While the active biofilm gains matter due to growth and attachment, and the inert biofilm gains matter by attachment and decay of the active biofilm, the inorganic solids film gains matter from attachment and precipitation of calcium carbonate. The rate of calcium carbonate precipitation is empirically derived yield coefficient (VanGulck et al. 2003). The resulting rate of calcium removal is used in the reaction term for calcium and in the calculation of the flux of calcium carbonate onto the porous media. Similar to the biofilms, by knowing the flux onto the surface and an estimate of density, the thickness added can be found.

Porosity and hydraulic conductivity

BioClog represents the porous media by assuming each element contains packed spheres of uniform diameter, onto which the films are attached. The density of the packing determines the clean media porosity. Given the total film thickness in the element, the porosity and specific surface are calculated geometrically based on Cooke and Rowe (1999). For each time step, the calculation of new film thicknesses then gives rise to new porosity and specific surface values.

Published correlations of clog matter quantity (i.e., mass, volume, porosity decrease) to hydraulic conductivity are scarce. For this reason, the hydraulic conductivity \((k)\) of each element is dependent on porosity \((n)\) alone, using an exponential relationship

\[ k = A_k e^{b_k n} \]

where coefficients \(A_k\) and \(b_k\) are estimated by regression analysis of measured data (see VanGulck and Rowe 2004;
Cooke et al. 2005b), where data is available. In the absence of data to the contrary, isotropic conditions are assumed, thus $k_x = k_z = k$.

Figure 3 shows the relationship between hydraulic conductivity and total porosity obtained from experiments examining the effects of clogging of columns permeated with synthetic or real landfill leachate. Also shown is the linear regression line (eq. [14]) and the associated parameters for this data as well as the relationships assumed for three hypothetical sands and the waste examined in the field demonstration case to be discussed later. It is assumed here that severely clogged porous media maintains some secondary porosity, and a hydraulic conductivity of $1 \times 10^{-8} m/s$ is adopted for the demonstration case as minimum $k$. This relationship between porosity and hydraulic conductivity could be readily replaced with other suitable equations when there is sufficient data to justify an alternative relationship.

**Issues relevant to variable mesh technique and clogging**

**Remapping element and node properties**

The mesh configuration changes when the surface elevation of the leachate mound changes (e.g., rises because of a reduction in hydraulic conductivity due to clogging) or when nodes are added or subtracted from the mesh. The consequent adjustment in nodal positions causes a change in element location and shape, but does not represent movement of the clog properties that were associated with the previous location of the element (since the clog matter is attached to the porous media, which remains static). Thus the element properties associated with the old configuration are “remapped” onto the elements in the new configuration. Since all other clog properties ($n, A, k$, etc.) can be derived from the five film thicknesses, only the thicknesses must be remapped from previous values as described by Cooke (2007). The porosity, specific surface, and hydraulic conductivity can then be computed for the elements based on the new film thicknesses. Details regarding the effect of layer interfaces (such as between a sand and gravel layer) on the movement, addition, and subtraction of nodes is explained in more detail in Cooke (2007).

**Maintaining adequate element aspect ratios**

To minimize numerical errors, the aspect ratio of the elements (the ratio of maximum to minimum dimension) should approach unity, and experience has shown that aspect ratios greater than 5 should be avoided (Anderson and Woessner 1991). To keep the number of nodes to a practical limit, the finite element mesh of a typical field case often consists of elements with horizontal dimensions larger than the vertical dimensions, but the vertical dimensions are variable over time. For this reason, after the model solves for the surface node elevations at time $t + \Delta t$, the element dimensions are subjected to criteria checks controlled by input parameters. If elements are deemed to have a vertical dimension that is too large relative to the horizontal dimension, the mesh may be regenerated with an additional row of...
Nonsteady drainage model for early times

If the drainage layer is coarse granular material (e.g., coarse gravel with \( k = 10^{-1} \) m/s), the saturated leachate mound thickness is very small (e.g., a maximum of 0.2 mm for a 30 m drainage path at a slope of 1% and infiltration rate of 0.2 m/year) at early times before there is significant accumulation of clog matter. Under these conditions, it was deemed impractical to model the mound using a 2D mesh because too many elements would be required to maintain adequate element aspect ratios. Considerable time may be required for clogging to cause the mound to rise to levels that can be adequately modelled in two dimensions. For these reasons, an approximate nonsteady mounding model was implemented that could be used to model early operation times until the time when the mound has risen to heights at which the 2D model can take over.

For this method, the shape of the surface is calculated based on the governing equation and boundary conditions described by McEnroe (1989) for nonsteady drainage of leachate on sloped barriers in landfill covers and liners. A single hydraulic head, and thus mound height, is calculated at each specified \( x \)-direction nodal spacing along the length of the flow path. The entire saturated zone is considered a single homogeneous unit with respect to species concentrations and clogging processes. Clogging calculations are performed only once per time step. Similar to the full 2D method, an iterative method is used each time step to maintain a balance between mound volume change, distribution of clog volume, and hydraulic conductivity.

This solution method allows accurate calculation of mound heights and flow velocities and reasonable computational speed while sacrificing spatial variability of clog properties when the mound is very small. Once the mound height is sufficient, as determined by a user-supplied maximum mound thickness, the model will switch to the full 2D method. Details regarding the implementation of this solution are given by Cooke (2007).

Model demonstration: Problem definition

To illustrate the use of the model, the results of a reasonably simple hypothetical field case will be presented. For brevity, not all input parameters will be discussed (all parameters not discussed in detail are well described by Cooke et al. (2005a) and the interested reader is referred to that paper for additional details). The system is illustrated in Fig. 4. The hypothetical field case being modelled is a sand drainage layer with a uniform thickness of 0.3 m, sloped at 1% to collection pipes spaced 40 m apart in a sawtooth arrangement (thus a drainage length of 20 m is modelled). Three cases using different sand properties, as listed in Table 1, are modelled. In case 1 the sand has an initial hydraulic conductivity, \( k_0 \), of \( 1 \times 10^{-4} \) m/s and grain size, \( d_g \), of 1.0 mm;
in case 2, \( k_0 = 1 \times 10^{-5} \text{ m/s} \) and \( d_g = 0.075 \text{ mm} \); and in case 3, \( k_0 = 1 \times 10^{-3} \text{ m/s} \) and \( d_g = 2.0 \text{ mm} \) (approximately representing medium, fine, and coarse sands, respectively). Each of the drainage materials has a different relationship between \( n \) and \( k \), as defined by \( A_k \) and \( b_k \) in Table 1 and shown in Fig. 3.

At the downstream end, the perforated collection pipe is assumed to have a diameter of 0.2 m and 5 mm of standing water, thus this boundary is modelled as a 0.2 m opening with 0.005 m specified head (A–B) and 0.195 m potential seepage face (B–C). The solute transport boundary condition is free for the open zone (A–C). The remainder of the boundary (\( Z > C \)) is zero flow and zero solute flux. A constant infiltration rate of 0.2 m/year is applied to the surface boundary (specified flux). For solute transport, this boundary is specified total solute flux. The upstream side boundary (\( Z > F \)) and bottom boundary (F–A) are both zero flow and zero solute flux. The leachate species concentrations entering the drainage layer from above are as given in Table 2.

The initial mesh used 5 rows and 245 columns of nodes (1952 triangular elements). The node spacing in the \( x \)-direction was 0.025 m at each end and gradually increased to 0.10 m for the majority of the mesh (from \( x = 1.8 \) to \( x = 18.2 \text{ m} \)). In preliminary testing to assess the effect of mesh refinement, this node spacing was shown to provide adequate modelling of flow and transport without further refinement. As the mesh rises an aspect ratio of 1.2 was used to guide the addition of nodes, as required. To aid the Cauchy source boundary condition, thin elements were employed along the phreatic surface with a thickness of 0.002 m. The choice of time step lengths is primarily controlled by surface convergence, and smaller time steps are generally required as time elapses. In case 1 the time step lengths were \( 2.5 \times 10^{-2} \text{ d} \) (0 to 7.5 years) and \( 1.25 \times 10^{-2} \text{ d} \) (7.5 to 10 years). In case 2 the time step length was \( 2.5 \times 10^{-2} \text{ d} \) from start to finish, and in case 3 multiple time step lengths were used, beginning at \( 2.5 \times 10^{-2} \text{ d} \) and ending at \( 2.0 \times 10^{-3} \text{ d} \).

Tables 3 and 4 list additional model parameters selected for these cases that are not specific to the granular drainage material (sands) and 2D application. These parameters have been discussed in detail in Cooke et al. (2005a, 2005b). A relaxation factor of 1.0 is employed for both the fluid flow and solute transport models, implying backward difference interpolation of the time derivative. Other numerical parameters are listed in Table 5. Based on the findings of Taylor et al. (1990), dispersivity increased with decreasing porosity.

### Table 3. Kinetics constants and diffusion parameters for the volatile fatty acids (parameters defined in detail by Cooke et al. 2005a).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Propionate</th>
<th>Acetate</th>
<th>Butyrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic constants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half maximum rate of concentration, ( K_s ) (mgCOD L(^{-1}))</td>
<td>4700</td>
<td>4700</td>
<td>4056</td>
</tr>
<tr>
<td>Maximum specific substrate utilization rate, ( q ) (mgCOD mgVS(^{-1}) d(^{-1}))</td>
<td>1.0</td>
<td>1.76</td>
<td>5.2</td>
</tr>
<tr>
<td>True yield coefficient, ( Y ) (mg VS mgCOD(^{-1}))</td>
<td>0.02</td>
<td>0.04</td>
<td>0.025</td>
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<tr>
<td>Biofilm decay coefficient, ( b_d ) (d(^{-1}))</td>
<td>0.02</td>
<td>0.018</td>
<td>0.02</td>
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<tr>
<td>Diffusion parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Substrate in fluid, ( D_f ) (cm(^2) d(^{-1}))</td>
<td>1.27</td>
<td>1.50</td>
<td>1.11</td>
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<tr>
<td>Substrate in film, ( D_i ) (cm(^2) d(^{-1}))</td>
<td>0.52</td>
<td>0.47</td>
<td>0.31</td>
</tr>
</tbody>
</table>

### Table 4. Clog and suspended solids parameters (parameters defined in detail by Cooke et al. 2005a).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clog matter parameters</td>
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<td>Calcium removal rate coefficient, ( Y_H ) (mg Ca(^{2+}) removed per mg H(_2)CO(_3) produced)</td>
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<tr>
<td>Variable biofilm density parameter, ( A_k )</td>
<td>247</td>
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<tr>
<td>Variable biofilm density parameter, ( B_k ) (mg VS cm(^{-3}))</td>
<td>72</td>
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<tr>
<td>Inorganic film density, ( X_{f,IS} ) (mg NVS cm(^{-3}))</td>
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<tr>
<td>“Other” precipitate ratio, ( f_{OP} )</td>
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<td>Fraction degradable by decay, ( f_d )</td>
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<td>Suspended solids parameters</td>
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<td>Active and inert diameter (cm)</td>
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<td>Inorganic particles density (cm)</td>
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<tr>
<td>Inorganic particles density (mg NVS cm(^{-3}))</td>
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<td>Attachment and detachment</td>
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<td>Attachment model (Tien 1989)</td>
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<td>Shear detachment modifier</td>
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<tr>
<td>Growth detachment modifier</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Table 5. Numerical and 2D specific parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical settings</td>
<td></td>
</tr>
<tr>
<td>Time step, ( \Delta t ) (d)</td>
<td>Variable</td>
</tr>
<tr>
<td>Relaxation factor, ( \omega )</td>
<td>1.0</td>
</tr>
<tr>
<td>Substrate convergence tolerance, ( \varepsilon )</td>
<td>0.001</td>
</tr>
<tr>
<td>Surface convergence tolerances, ( f_{IS}, f_{IZ} )</td>
<td>0.05</td>
</tr>
<tr>
<td>Surface extrapolation multiplier, ( E )</td>
<td>1.0</td>
</tr>
<tr>
<td>Limit surface node movement</td>
<td>No</td>
</tr>
<tr>
<td>Element orientation</td>
<td>All right oriented</td>
</tr>
<tr>
<td>Limits</td>
<td></td>
</tr>
<tr>
<td>Minimum hydraulic conductivity, ( k_{min} ) (m/s)</td>
<td>( 1 \times 10^{-8} )</td>
</tr>
<tr>
<td>Minimum saturated thickness, ( \gamma_{min} ) (cm)</td>
<td>0.3</td>
</tr>
<tr>
<td>Dispersivity</td>
<td></td>
</tr>
<tr>
<td>Longitudinal dispersivity, ( \alpha_L ) (cm)</td>
<td>( \alpha_L = \alpha_{L,0} \left( \frac{t}{E} \right)^{b_L} )</td>
</tr>
<tr>
<td>Initial longitudinal dispersivity, ( \alpha_{L,0} ) (cm)</td>
<td>10</td>
</tr>
<tr>
<td>Equation parameter, ( b_L )</td>
<td>-1.74</td>
</tr>
<tr>
<td>Transverse dispersivity, ( \alpha_T ) (cm)</td>
<td>( \alpha_T = c_a \alpha_L )</td>
</tr>
<tr>
<td>Equation parameter, ( c_a )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Note:** Film density \( X_{f,IS} \) is mass of nonvolatile solids (NVS) per volume of inorganic solids.
The transverse dispersivity is assumed to be equal to the longitudinal dispersivity.

**Model demonstration: Results**

The modelling was performed until the maximum thickness of the mound exceeded 0.30 m, as this is often regarded as a maximum allowable height for design purposes and also because at this point the mound will begin to be affected by the properties of the waste, which are often more uncertain than those of the drainage media. This thickness was reached just before 10, 0.75, and 32 years for cases 1, 2, and 3, respectively, demonstrating the significant effect that particle size can have on mound height and on the time the head may stay below 0.3 m. A comparison will be made to the Giroud equation (Giroud et al. 1992), and then the results for case 1 (base case) will be presented in detail, followed by the mound progression and porosity data for cases 2 and 3.

The cases examined next are intended to illustrate the application of the model. They only consider one set of parameters (other than the difference in initial particle size and hydraulic conductivity of the sand). Clogging will vary depending not only on these parameters but also on the influent concentrations, kinetic parameters etc. Space does not permit a discussion of these issues in this paper. Rather, the objective here is to present a model that can examine these factors. The effect of these parameters will be discussed in a subsequent paper.

**Comparison to the Giroud equation**

To compare the model results for each case with results from the Giroud equation (Giroud et al. 1992), the three runs were first performed without clogging, and for case 2, without a waste layer. The calculated maximum mound thicknesses developed over the first 2 years are shown in Fig. 5. Without clogging, the mound rises rapidly and reaches the steady-state thickness predicted by the Giroud
equation (0.077, 0.38, and 0.011 m for cases 1, 2, and 3, respectively) within a few months for cases 1 and 3 and within about 2 years for case 2. Also shown in Fig. 5 is the calculated mound development when clogging is considered.

Case 1 with clogging

For case 1 (medium sand), the mound shape at 2 year intervals is shown in Fig. 6. After an initial rapid rise (see Fig. 5) it can be seen from Fig. 6 that the mound rises at a fairly constant rate for subsequent times. The typical maximum design head of 0.3 m is reached after a little less than 10 years. The calculated porosities within the mound are shown at 5 and 10 years in Fig. 7. After 10 years, porosity has decreased from the initial 0.35 to as low as 0.25 at the downstream end, with the average being 0.31. While clogging was most extensive at the downstream end, which is to be expected because of the relatively large mass loading, there was also a clogged region at the upstream end of the slope. This was likely caused by the low flow and thus long residency time and decreased shear detachment at this end. Also, there was a slight increase in porosity near the base, very likely due to the removal of substrate (VFAs) and suspended solids near the surface and hence there was less mass loading near the base (except near the collection pipe).

Contours of the calculated hydraulic conductivities within the mound at 5 and 10 years are shown in Fig. 8. After 10 years of clogging, the hydraulic conductivity dropped from an initial $1 \times 10^{-4}$ m/s to a low of $2.6 \times 10^{-6}$ m/s at the downstream end near the collection pipe. The average within the mound was $1.9 \times 10^{-5}$ m/s. Assuming a homogeneous drainage layer with this average hydraulic conductivity, the Giroud equation predicts a maximum mound thickness of 0.25 m compared to the 0.31 m calculated by BioClog assuming a heterogeneous system.
Figures 9, 10, and 11 show the calculated thicknesses of active, inert organic, and inorganic films, respectively, in the mound at 5 and 10 years. The active films (Fig. 9) were the thinnest of the films and generally decreased in thickness with distance from the surface. According to the thickness of the three types of films after 5 and 10 years, the inert organic film dominated the clog matter, followed by the inorganic film. The distributions of the inert, inorganic, and total film thicknesses (Figs. 10, 11, and 12, respectively) were all similar. The largest films were located at the down-
stream end where film thicknesses increased within a relatively short 1 m region – likely caused by the relatively high mass loading at this end. In the remainder of the mound, there was a slight increase in film thickness near the surface and upstream end, likely controlled by the proximity to the source of growth substrate (VFAs) and suspended solids for attachment. The latter influences, coupled with the rising mound and thus greater distance to the surface, can be seen to cause a general decrease in active film thickness between 5 and 10 years. The reduced active film
thickness near the base, over time, leads to reduced rates of calcium carbonate precipitation. Understandably, the distributions of the dominating inorganic and inert films (and thus also total film thickness) follow similar patterns to that of porosity and hydraulic conductivity.

The calculated concentrations of acetate (Fig. 13) and cal-

Fig. 12. Case 1 – total thickness of clog films (mm) within the mound after 5 and 10 years.

Fig. 13. Case 1 – acetate concentrations (mg/L) within the mound after 5 and 10 years.
Fig. 14. Case 1 – calcium concentrations (mg/L) within the mound after 5 and 10 years.

Fig. 15. Case 2 (fine sand, $k_o = 10^{-5} \text{ m/s}$) – mound surface profiles at intervals of 0.25 years for fine sand.

Fig. 16. Case 3 (coarse sand, $k_o = 10^{-3} \text{ m/s}$) – mound surface profiles at intervals of 8 years for coarse sand.
In the majority of the mound, the acetate has been depleted to concentrations of 800 to 1200 mg/L and calcium to concentrations of 650 to 800 mg/L. Low concentration zones appear upstream. It should be noted that significantly greater calcium depletion (and inorganic clog accumulation) would result if a larger value for the calcium removal rate, $Y_H$, was employed (such as $Y_H = 0.116$ as reported by VanGulck et al. 2003).

**Results for cases 2 and 3 with clogging**

The mound progression for case 2 (approximating a fine sand, $k_0 = 1 \times 10^{-5}$ m/s and $d_g = 0.075$ mm) is shown in Fig. 15 at intervals of 0.25 years. Because of the low initial hydraulic conductivity the leachate mound would exceed 0.3 m even if there was no clogging. With clogging, the top of the drainage layer was reached before 0.75 years. The rate of rise in this case was gradually decreasing, but is impacted to a greater extent by the properties of the waste. Starting at an initial porosity of 0.33, the average was 0.32 and the low (occurring at the downstream end) was 0.28 after 1 year. With the mound reaching the waste quickly, relatively little clogging occurred during the time modelled.

Due to the considerably slower clogging rate associated with a drainage material of larger grain size and hydraulic conductivity of case 3 (approximating a coarse sand with $k_0 = 1 \times 10^{-3}$ m/s and $d_g = 2.0$ mm), it required 32 years (Fig. 16) for the top of the drainage layer to be reached compared to 10 years for the fine and medium sand. The rate of rise was relatively constant after the initial period. Figure 17 shows the predicted porosities in the mound after 16 and 32 years. The initial porosity was 0.37. After 32 years, the average porosity in the mound was 0.24 and the lowest porosity, at the downstream end near the pipe, was 0.17.

**Conclusions**

A 2D model for predicting the change in porosity and hydraulic conductivity of drainage layers and the subsequent increase in leachate head on the liner as a result of clogging by biological and physical processes in landfill leachate collection systems has been described. The model includes flow modelling with a dynamic surface, solute transport of the nine species most responsible for clogging, the fate of these species, and the corresponding increase in clog material volume. Porosity and hydraulic conductivity both change spatially and temporally within the leachate mound. The demonstration of the model using typical field parameters shows that, depending on the nature of the material used in the drainage layer, clogging can cause a decrease in hydraulic conductivity that can cause the leachate mound to exceed the design values in periods ranging from about 1 to 32 years for sand with initial hydraulic conductivities of $10^{-5}$ to $10^{-3}$ m/s. The model provides an opportunity to explore the effect of different drain spacing, drainage materials, and leachate characteristics.

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References


