Theoretical solutions for axial deformation of drilled shafts in rock

R. K. Rowe
Geotechnical Research Centre, Faculty of Engineering Science, The University of Western Ontario, London, Ont., Canada N6A 5B9

AND

H. H. Armitage
Golder Associates, 500 Nottinghill Road, London, Ont., Canada N6K 3P1

Received May 20, 1986
Accepted October 16, 1986

A theoretical examination of a number of factors affecting the behaviour of drilled piers in soft rock is presented. Firstly, the fundamental difference between tests commonly used for determining the peak average side shear resistance along a socketed pier is discussed. Secondly, the effect of interface strength parameters, dilatancy, and the relative Young's modulus of the pier and rock upon the average mobilized side shear resistance is examined. Thirdly, a series of theoretical solutions are presented in the form of design charts to provide a simple means of estimating the load—deflection response of piers both before and after full slip has developed along the pier shaft. Finally, the effect of weak horizontal seams adjacent to the pier is considered.

Key words: drilled shafts, piers, piles, rock, theory, settlement, displacement, analysis.

Un examen théorique d'un certain nombre de facteurs influençant le comportement de pieux forés dans la roche molle est présenté. Premièrement, la différence fondamentale entre les essais utilisés couramment pour déterminer le pic moyen de résistance au cisaillement latéral le long de l'enveloppe d'un pieu est discutée. Deuxièmement, l'effet des paramètres de résistance à l'interface, de la dilatance, et du module de Young du pieu et de la roche, sur la moyenne de la résistance au cisaillement mobilisée sur les côtés est examiné. Troisièmement, une série de solutions théoriques est présentée sous la forme d'abaque pour fournir un moyen simple d'estimer la réaction charge—déflexion des pieux tant avant qu'après qu'un glissement complet ait produit le long du fût. Finalement, l'effet de couches faibles horizontales adjacentes au pieu est considéré.

Mots clés : puits forés, pieux, roche, théorie, tassement, déplacement, analyse.


Introduction

Large foundation loads are often supported by drilled piers or piles socketed into rock. In the past, the design of these piers/piles has been carried out using empirical values of allowable stresses on the base and the shaft. More recently, various researchers have advocated the use of higher allowable shear resistance in conjunction with the use of elastic theory for socketed pier design (e.g. Osterberg and Gill 1973; Ladanyi, 1977; Pells and Turner 1979; Horvath 1980; Kulhawy and Goodman 1980; Williams and Pells 1981).

There is considerable evidence to suggest that both approaches are safe. However, they may be too safe. As design column loads and the cost of constructing rock sockets both increase, there is considerable motivation for the development of new design approaches that will avoid the increasing costs of unnecessary overdesign while guarding against the consequences of underdesign.

This need has been recognized by a number of authorities around the world who have funded large-scale field tests (e.g. in Canada the Department of Supply and Services Canada/National Research Council of Canada funded work reported by Horvath (1980) and Horvath et al. (1983); in Australia the Commonwealth Department of Construction funded work reported by Pells et al. (1980); and the Country Roads of Victoria funded work reported by Williams (1980)). These investigations have provided a wealth of field data indicating that socketed piers/piles may be safely designed to carry loads well in excess of those commonly adopted. However, these field investigations have not been adequately complemented by theoretical analysis, and up to this point in time there is still no thorough design procedure that takes full advantage of the increased pier/pile capacity indicated by these field testing programmes.

The present paper describes a theoretical examination of a number of factors affecting the behaviour of socketed piers/piles. Firstly, the fundamental difference between tests commonly used for determining the peak average side shear resistance along a socketed pier is discussed. Secondly, the effect of interface strength parameters, dilatancy, and the relative Young's modulus of the pier and rock upon the average side shear resistance mobilized in end-bearing piers is examined. Thirdly, a series of theoretical solutions are developed that provide a means of estimating the load—deflection response of a pier both before and after full slip is developed along the pier shaft. Finally, the effect of weak horizontal seams adjacent to the pier is considered.

Based on the theoretical solutions presented in this paper, the authors have developed a new procedure for the design of socketed piers in weak rock. This procedure is described and illustrated with reference to observed field behaviour by Rowe and Armitage (1987a, b).

Principal assumptions and numerical procedure

This paper is concerned with the final equilibrium settlements (neglecting creep) at the head of a concrete pier of diameter \( D \) and socket length \( L \) (see Fig. 1).

It is assumed that the pier is an isotropic, homogeneous
elastic solid with a Young’s modulus $E_p$ and Poisson’s ratio $\nu = 0.15$. The rock is considered to be isotropic and layered. The moduli of the rock adjacent to and beneath the socketed pier are $E_a$ and $E_b$, respectively. The modulus of soft horizontal seams encountered in the rock mass is $E_s$. The Poisson’s ratio for all rock material is 0.3.

The theoretical analyses were performed using an elastoplastic, axisymmetric finite element program (ROSOC) based on the general method of analysing rock/soil-structure interaction problems described by Rowe et al. (1978). A typical finite element mesh using eight-noded isoparametric elements is shown in Fig. 2. The method of analysis makes provision for plastic failure within the rock as well as for slip at a cohesive-frictional pier-rock interface such as that indicated by the dual nodes (one each side of the interface) in the insert to Fig. 2. Details regarding the method of analysis are given by Rowe and Pells (1980). In some cases, strain softening and/or dilatancy of this interface was considered as described in the following section. Strain softening or brittle failure within the general rock mass was not considered.

To check the suitability of the finite element mesh adopted, a comparison was made with results from (a) several different finite element mesh arrangements where the degree of mesh refinement was varied and (b) published numerical solutions (e.g. Mattes and Poulos 1969; Pells and Turner 1979; Donald et al. 1980). On the basis of these checks, it is considered that the basic finite element mesh is adequate and that the solutions presented in this paper are accurate to within $5\%$-10%.

Methods used to evaluate side shear resistance

Field tests to determine average shear resistance take a number of forms in terms of both the socket arrangement and the method of loading. Frequently, end-bearing resistance is eliminated by casting the concrete socket above the base of the drilled hole. Thus, the average side shear resistance is simply the applied load divided by the surface area of the concrete socket and this socket is referred to herein as a “side shear socket.” Alternatively, end bearing is permitted and instrumentation is provided so that the proportions of load carried in end bearing and side shear can be determined.

For sockets where end bearing has been eliminated (i.e., a “side shear socket”), it is common to perform a displacement-defined test where the load is either increased or decreased (as necessary) to achieve a specified increase in displacement. This approach allows convenient determination of both the peak and residual side shear values since the load can be reduced to follow the postpeak response.

A load-defined test (where the load is increased and the displacement is measured) is commonly used in association with end-bearing (“complete”) sockets. If the load transferred in end bearing is measured, then the peak and residual side shear values may be deduced by calculating the proportion of load carried in side shear.

Displacement-defined tests on side shear sockets tend to be cheaper and easier to perform (since lower loads are required) than load-defined tests on end-bearing sockets. However, for many (but not all) practical situations, the latter test more truly represents the real situations and so it is important to note that there are fundamental differences between the nature of the two approaches and that this may significantly affect the deduced side shear values, particularly for relatively smooth sockets.

For a given mobilized average side shear, the applied load on an end-bearing (“complete”) socket will be greater (and hence Poisson’s effects will be greater) than for a side shear only socket. Furthermore, the presence of a void beneath the concrete plug will influence the stresses along the side of the plug, resulting in a stress distribution different to that which occurs for an end-bearing socket. Thus, even with the same loading method, some differences would be expected between side shear only and complete socketed piers. The difference between load techniques commonly adopted in these cases will accentuate the difference in observed behaviour, especially for sockets exhibiting strain softening.

In the displacement-defined test on a relatively smooth isolated socket, the applied load will decrease as softening occurs...
at the interface (e.g. as the concrete—rock bond is broken). The reduction in applied load reduces the normal stress along the interface, which, in turn, also reduces the frictional component of the shear resistance, resulting in a very brittle response.

In a load-defined analysis on a relatively smooth end-bearing socket, the applied load remains constant or increases as softening occurs at the concrete—rock interface, with the additional load being carried in end bearing. This maintenance or increase in applied load maintains, or increases, the normal stress along the interface thereby increasing the frictional component of the available shear resistance and reducing the brittleness of the response.

The greatest discrepancy between the peak and residual side shear resistance deduced from different tests will occur for very smooth sockets. Increasing roughness of the socket (reducing brittleness) will reduce this discrepancy.

A theoretical study of the effects of a number of fundamental parameters upon the response of side shear sockets subjected to a displacement-defined loading has been reported by Rowe and Pells (1980). In this paper, attention will be primarily directed to the behaviour of complete socketed piers in a load-defined test.

**Fundamental factors influencing side shear resistance**

The average side shear resistance values \( \tau_r \) deduced from a series of tests in Melbourne mudstone (Williams et al. 1980) and Hawkesbury sandstone (Pells et al. 1980) are summarized in Fig. 3. Also shown is the best-fit correlation between side shear resistance and uniaxial compressive strength deduced by Rowe and Armitage (1984) from an examination of over 200 load tests. From the data points shown, it is evident that there is a wide scatter of results even for a particular test method and rock type. Some statistical variation in results is to be expected; however, it is the objective of this section to illustrate that there are also deterministic factors contributing to this variability in mobilized side shear resistance. These factors include the stiffness of the pier relative to the rock \( (E_p/E_r) \), the length-to-diameter ratio of the pier \( (L/D) \), the initial stresses in the rock, the rock—concrete interface strength properties, and the socket roughness. To examine the effect of these parameters, analyses were performed using fundamental rock properties appropriate for Sydney sandstone and Melbourne mudstone. Although these fundamental parameters are usually not available for the design of most sockets, this fundamental analysis is intended to provide guidance in the interpretation and application of field test data to the design of socketed piers.

**Theoretical model — fundamental interface behaviour**

The shear strength, \( \tau \), at a point along the interface was modelled in terms of a Mohr—Coulomb failure criterion:

\[
\tau = c + \sigma_n \tan \phi
\]

where \( \sigma_n \) is the normal stress at the interface, \( c \) is the interface “adhesion”, and \( \phi \) is the interface friction angle. It was assumed that \( c \) and \( \tan \phi \) degrade linearly with relative displacement between two sides of the rupture from peak values \( (c_0, \tan \phi_0) \) to residual values \( (c_r, \tan \phi_r) \) at a relative displacement \( \delta_r \). Roughness of a socket was modelled in terms of a dilatancy angle \( \psi \) (Davis 1968) and a maximum dilation \( \delta_d \).

**Effect of roughness, modulus ratio, and geometry**

Considering firstly the Hawkesbury sandstone examined by Pells (1977) and Pells et al. (1980), analyses were performed for the interface strength parameters determined by Pells (viz. \( c_p = 4 \) MPa, \( \phi_p = 39^\circ \), \( c_r = 0 \), \( \phi_r = 36^\circ \), \( \delta_r = 0.003D \)) for both a perfectly smooth socket \( (\psi = 0) \) and a moderately rough socket \( (\psi = 20^\circ , \delta_r = 0.001D) \). The theoretical results are summarized in Fig. 4 for a typical range of modulus ratio \( E_p/E_r \). The experimental results obtained by Pells et al. for straight smooth-sided sockets (grooves or indentations less than 1 mm deep) are also shown in Fig. 4a while the two results for slightly rougher sockets (grooves of depth 1—4 mm, width greater than 2 mm at spacing 50—200 mm) are shown in Fig. 4b.

The response of the smooth-sided sockets is quite brittle and the residual average side shear values are considerably lower than the peak values. The calculated and observed peak average shear values are in generally good agreement. The calculated residual average shear values are less than the observed values, probably because the actual sockets were not perfectly smooth. Although there was some variation with the \( L/D \) ratio, the effect was not particularly significant for these smooth sockets.

Increasing roughness of the socket increases both the peak and residual average side shear values and reduces the brittleness of the response. The stiffer the rock \( (i.e., \text{the lower the ratio } E_p/E_r) \), the larger will be the normal stress developed at a given level of dilation of the interface. Thus, the effect of roughness is considerably greater for \( E_p/E_r = 25 \) than it is for \( E_p/E_r = 100 \). For similar reasons, the effect of the length-to-diameter ratio \( (L/D) \) is also more significant for lower values of \( E_p/E_r \) \( (i.e., \text{stiffer rock}) \).

In general, relatively little is known about the relationship between rock mass modulus, roughness, and dilatancy at the interface. However, the work by Williams (1980) does provide some guidance regarding the parameter for very soft rock (Silurian mudstone). Considering a rock with an uniaxial compressive strength of 0.72 MPa, \( c_p = 0.2 \) MPa, and \( \phi_p = 32^\circ \), analyses were performed for a range of modulus ratios \( (E_p/E_r) \) corresponding to different levels of weathering and jointing in weak rock. The interface strength parameters \( (c_p = 0.2 \text{ MPa}, \phi_p = 32^\circ , c_r = 0, \phi_r = 21^\circ) \) were taken to be independent of rock stiffness. The roughness \( (\psi, \delta_d) \) parameters were selected from consideration of the experimental data and varied with rock stiffness as indicated in Fig. 5.
to 0.82 (100 ≤ E_p ≤ 500). The average ratio determined from William’s tests was 0.8, although the range of values was wider than calculated, being from 1 to 0.45 for a similar range in modulus values.

The variation in average side shear resistance with modulus ratio is of practical significance if, as is often the case, the side shear resistance is to be related to the uniaxial compressive strength of rock using empirical correlations. For a rock with a given uniaxial compressive strength of typical (intact) samples, the mass modulus may vary appreciably depending on the degree of jointing and the presence of seams. Figure 5 illustrates the effect of mass modulus (relative to an intact modulus) upon the average side shear resistance. Both the theoretical curve (derived from this fundamental analysis) and the experimental curve (from Williams 1980) show that the mobilized side shear resistance may be substantially reduced by jointing. This result serves as a warning that empirical correlations developed for tightly jointed rock cannot be used directly for highly jointed rock, which may have a high intact strength but low mass modulus.

The influence of high horizontal rock stress

In the foregoing analyses, it was assumed that the initial horizontal stresses at the pier-rock interface were negligible relative to the normal stresses due to Poisson’s effect and dilation of the interface. The assumption is likely to be valid for most practical cases; however, stress measurements around the world have indicated that high horizontal stresses may be encountered near the surface of a rock mass (see e.g. Palmer and Lo 1976). Under certain circumstances, involving a time-dependent rock mass subjected to high horizontal stresses, one may expect the development, with time, of appreciable horizontal stresses between the pier and the rock after casting of the socket. Analyses performed for this case indicate that, as might be expected, increasing horizontal stress increases the magnitude of the peak and residual mobilized shear. The magnitude of this effect depends on the time between excavation, concreting, and testing of the socket, and the initial horizontal stresses within the strata. Thus, the effect of high horizontal stresses upon the mobilized side shear resistance may be quite variable and should not be relied upon unless the horizontal stresses and time effects are well known. Similarly, values of side shear resistance deduced from load tests in rocks with high horizontal stresses should be used with caution unless these stresses are accounted for properly.

Settlement factors and load distribution

Elastic solutions (e.g. Pells and Turner 1979; Donald et al. 1980) currently used to estimate socket settlement and the proportion of load carried to the base of the socketed pier were developed assuming full bond between the pier and the rock (i.e., no slip). An examination of the elastic stress distribution along the socket for typical situations indicates that in many cases the elastic shear stress exceeds the available interface shear strength and slip would be expected along the socket. In fact, this type of behaviour has long been recognized in the design of piers in clay (see e.g. Whitaker and Cooke 1966; Burland et al. 1966; Poulos 1972; Poulos and Davis 1980) where the deformations required to mobilize base resistance may be sufficient to cause full slip along the pier at working loads. This has also been recognized by Kulhawy and Goodman (1980), who show full bond and frictional transfer curves.
for rock based on assumptions that allow the development of some simple equations.

Recognizing the likelihood that some slip will occur at the pier—rock interface, it would seem reasonable to consider this possibility during the design procedure. Accordingly, finite element solutions will be presented to permit the determination of

- the pier head settlement and load carried to the base under elastic (no slip) conditions;
- the load at which full slip occurs and the corresponding pier head settlement;
- the pier head displacement for any given load following full slip.

The behaviour of a socketed pier up to and including collapse could be predicted using the approach described in the previous sections. However, in practice, there will generally be insufficient data to justify such a detailed analysis. Thus, for purposes of design, it is convenient to adopt a simplified approach in which it is assumed that

- any failure occurs at the pier—rock interface;
- the available shear resistance \( \tau_s \) at the pier—rock interface is constant along the socket length and is taken to be the average side shear resistance determined either from a fundamental analysis (see the previous section) or, more probably, from field load test results and (or) empirical correlations (see Rowe and Armitage 1984), with possible adjustments for the effect of mass modulus as indicated by Fig. 5);
- the rock adjacent to the pier is homogenous with a mass modulus \( E_r \) and \( v = 0.3 \) (seams will be considered in the next section);
- the rock beneath the pier base is homogenous with a mass modulus \( E_b \) and \( v = 0.3 \) (nonlinearity of this rock may be approximately considered by use of an appropriate secant modulus).

**Results of the finite element analysis**

The results in this section were obtained for a range of pier modulus to rock modulus \( (E_p/E_r) \) values and base modulus to side rock modulus \( (E_b/E_r) \) values. For specific values of \( E_p/E_r \), \( E_b/E_r \), and \( L/D \), the results are presented in terms of the dimensionless displacement, \( I = pE_rD/Q_t \), corresponding to a given ratio of load \( Q_b/Q \), being carried to the base of the socketed pier, where (see Fig. 1) \( Q \) is the applied load on the pier at the rock surface; \( Q_b \) is the load carried to the base of the pier; \( D \) is the pier diameter; \( L \) is the pier length; and \( p \) is the average pier head displacement (an average of the centre-line and edge displacements).

Typical solutions for \( E_p/E_r = 1.0 \) and \( E_b/E_r \) of 25 and 100 are presented in Figs. 6 and 7 (a complete set of design charts is given in the Appendix). These figures summarize the load—settlement response for piers with \( L/D \) ratios from 1 to 10 for a wide range of conditions.

The elastic solution shown as the lower dotted contour in Figs. 6 and 7 represents the limiting case of minimum displacement and minimum load carried to the base of the pier. Increasing slip along the pier results in increased displacements and an increased proportion of load carried to the base of the pier. The upper dotted line corresponding to \( \tau_{fl}/\tau_s = 1 \) represents the case of full slip. Thus, given the rock parameters and socket geometry (i.e., \( E_p/E_r \), \( E_b/E_r \), \( \tau_s \), \( L/D \)) these results provide a means of constructing the entire load—settlement response for a socket. For example, the load \( Q \) required to cause full slip may be determined for a given \( L/D = 4 \), \( E_p/E_r = 25 \), and \( E_b/E_r = 1 \) from Fig. 6 by moving vertically upwards from the known value of \( L/D \) until the dotted “full-slip” line is
reached. The dimensionless displacement \( I_n = \rho E_s D/Q_t \) can then be obtained by interpolation between the elastic value of \( I (\sim 0.3) \) and the contour for 0.35 giving \( I_n = 0.315 \). Moving horizontally from the point of intersection with the full-slip line gives the ratio of load carried to the base \((Q_b/Q)_b = 0.16\). Thus, at the onset of full slip, only 16% of the total load is carried to the base. At full slip, the load carried in shear \( Q_s \) is known, viz.,

\[ 2 \] \( Q_s = \pi D L \tau_f \)

and since

\[ 3 \] \( \frac{Q_b}{Q_t} = \frac{Q_t - Q_s}{Q_t} = 1 - \frac{Q_s}{Q_t} \)

the load \( Q_t \) required to cause full slip

\[ 4 \] \( Q_t = Q_t/(1 - (Q_b/Q)_b) \)

may be calculated. The corresponding average pier head displacement is then given by the expression

\[ 5 \] \( \rho_{hs} = \frac{Q_t}{E_s D} I_n \)

where \( Q_t, E_s, D, \) and \( I_n \) are now all known.

Prior to full slip, the load–settlement response can be estimated for a given level of slip (e.g. \( \tau_{wsl}/\tau_f \)) by moving vertically upwards from the known value \( L/D \) until the appropriate partial-slip line is reached. For example, with \( L/D = 4 \) and \( E_p/E_r = 25 \) (Fig. 6), the values of \( I \) and \( Q_b/Q_t \) corresponding to \( \tau_{wsl}/\tau_f = 0.9 \) are 0.31 and 0.15 respectively.

The load carried in side shear when only partial slip has occurred is

\[ 6 \] \( Q_{ps} = (\tau_{wsl}/\tau_f)\pi D L \tau_f \)

\[ = 0.9Q_s \quad \text{(in this case)} \]

The corresponding top load \( Q_t \) is then given by

\[ 7 \] \( Q_t = Q_{ps}/(1 - (Q_b/Q_t)) \)

and hence the displacement can be calculated for this value of \( Q_t \) from

\[ 8 \] \( \rho = \frac{Q_t}{E_s D} \)

In this case, it can be seen that there is very little difference between the elastic values of \( I, Q_b/Q_t \) and the values for \( \tau_{wsl}/\tau_f = 0.9 \). Thus for any applied load \( Q_t \) less than that calculated from \( 7 \) above, the response of the socket can be considered to be elastic.

An inspection of Fig. 7 indicates that here there are no curves for \( \tau_{wsl}/\tau_f < 1 \). In cases such as this, the pier may be regarded as exhibiting an essentially elastic response for values of \( Q_t \) less than or equal to that calculated from \( 6 \) and \( 7 \) for \( \tau_{wsl}/\tau_f = 0.9 \). The elastic values of \( I \) and \( Q_b/Q_t \) can be used for these cases.

**After full slip**, it is assumed that the load carried in side shear remains constant at the value given by \( 2 \) and hence the value of \( Q_b/Q_t \) for a given applied load \( Q_t \) is

\[ 9 \] \( \frac{Q_b}{Q_t} = 1 - \frac{\pi D L \tau_f}{Q_t} \)

The dimensionless displacement \( I \) may be read from the design charts for the specified value of \( Q_b/Q_t \) and \( L/D \) and hence from \( 8 \),

\[ \rho = \frac{Q_t}{E_s D} \]

Thus the results given in these figures provide a means of determining the load displacement behaviour for a given pier.

In the design of socketed piers, it may be argued that it is necessary to (a) limit settlements to some specified allowable value \( p_s \) and (b) ensure an adequate factor of safety against end-bearing failure. Provided that these conditions are satisfied, there would appear to be no fundamental reason for not allowing full slip along the socket. (Indeed, as previously noted, slip is likely to occur irrespective of whether it is considered in design.) Assuming that the possibility of full slip is acceptable and that the pier and rock properties \( E_p, E_r, \tau_f, D \) can be estimated, then the optimum length of pier required to limit deflection to the prescribed value \( p_s \) for a given load \( Q_t \) may be conveniently determined from the appropriate design chart as follows:

From equation \( 9 \),

\[ 10 \] \( \frac{Q_b}{Q_t} = 1 - \frac{\pi D L \tau_f}{Q_t} = 1 - 4\left(\frac{L}{D}\right)\frac{\tau_f}{q_t} \)

where \( q_t = Q_t/(\pi D^2/4) \) is the average applied pressure on the pier at the rock surface. Since the average side shear \( \tau_f \) and applied pressure \( q_t \) are known, the ratio of base load to top load
$Q_b/Q_i$ is a linear function of $L/D$. The optimal pier length is that which satisfies [10] and gives a dimensionless displacement

$$I_d = \frac{pL_D}{Q_i}$$

This may be determined graphically by plotting the straight line given by [10] on the appropriate design chart and finding the intersection between this line and the contour corresponding to the dimensionless displacement $I_d$; this then gives the required pier length. This pier automatically satisfies condition (a). The value of $Q_b/Q_i$ for this pier is read from the chart, and the base pressure, $q_b$, can then be calculated, viz.,

$$q_b = \frac{(Q_b/Q_i)Q_i}{(\pi D^2/4)}$$

and compared with the allowable end-bearing pressure. Provided that condition (b) is satisfied, the optimal pier length has been determined. The selection of design parameters and the use of these theoretical results to develop a design procedure are described by Rowe and Armitage (1987a, b).

When considering the results given in Figs. 6 and 7 (and the Appendix), it should be recognized that the equilibrium conditions for full slip are defined by [10]. Thus, for any specified applied load and rock/pier parameters, there is a unique (or no) value of $I = pL_D/Q_i$ corresponding to this particular geometry and load. Thus, the contours of $I$, the influence factor, do not correspond to a variation in displacement for a given load but rather to a variation in both $I$ and displacement where the actual load at any point along the contour must be determined using [10]. These contours reflect both the compressibility of the pier and rock, and the distribution of load along the pier. Consequently, the shape of the contours varies with modulus ratio $E_p/E_r$.

Figures 8 and 9 show the variability in load distribution ($Q_b/Q_i$) as a function of modular ratios $E_p/E_r$ and $E_o/E_r$, respectively, for a range of influence factors $I$. The load carried to the base of the pier for a given value of $L/D$ and $I$ increases significantly with increasing pier modulus ($E_p/E_r$) and increasing base modulus ($E_o/E_r$). These charts may be used to estimate the value of the dimensionless displacement $I$ (for given $Q_b/Q_i$) when the modulus ratios $E_p/E_r$ or $E_o/E_r$ do not
From Eq. [15]

Effect of horizontal seams on socketed pier behaviour

The behaviour of piers socketed into rock that is tightly jointed may be approximately predicted using the theory of the previous section in conjunction with an appropriate mass modulus. However, if the joints are filled with weathered material, particular consideration must be given to the effect of these weaker zones on the pier response.

In this section, the effect of contiguous horizontal seams located above the bottom of the socket will be examined. It is assumed that these seams have a modulus $E_s$ and a shear resistance $\tau_s$ at the pier–seam interface. The effect of the seam on pier head settlement and load distribution might be expected to be a function of the percentage of seams and their distribution along the pier, the relative seam modulus $E_s/E_r$, and relative $\tau_s/\tau_r$ (where $\tau_r$ is the average shear resistance of the pier–rock interface). Seams may be evenly distributed or may be concentrated over some portion of the pier. However, it is generally conservative to restrict attention to the situation where the seam is concentrated just above the base of the pier.

Results of the finite element analysis

For a given geometry and load, the proportion of base load $(Q_b/Q)$ and the settlement factor $I$ will both increase due to the presence of soft seams adjacent to the pier. Taking $S$ to be the proportion of seam along the socket length, experimental evidence (see e.g. Pells et al. 1980; Thorne 1980) would suggest that the effect of seams on the side shear resistance $\tau_s$ can be considered by

\[ \tau_s^* = S\tau_r + (1 - S)\tau_s, \]

where, in many cases, the shear resistance along the seam ($\tau_s$) may be neglected altogether. In determining the effect of seams on the mass modulus $E_r^*$, two approaches will be considered.

The simplest approach is to calculate an equivalent modulus $E_r^*$, where

\[ E_r^* = SE_r + (1 - S)E_s, \]

and hence the influence factor $I^*$, which reflects the effect of seams, is related to the “design” influence factor $I_d$ (i.e., neglecting seams) by

\[ I^*/I_d = 1 - S + SE_r/E_r. \]

The charts given in Figs. 6 and 7 (and the Appendix) can then be used together with $\tau_s^*/\tau_r$, and $[9]$ as described in the previous section.

To assess the potential errors associated with this simplified approach, a series of finite element analyses were conducted where various proportions of seams along the socket (with specified properties $\tau_s/\tau_r$, $E_s/E_r$) were directly modelled and the results of some of these analyses have been summarized in Figs. 10 and 11 in terms of the ratio $I^*/I_d$ deduced from the finite element analysis for different influence factors $I_d$. (In these figures, $I_d$ is the influence factor deduced from a homogeneous analysis for a given load level and $I^*$ is the influence factor corresponding to the same proportion of load carried to the base of the socket from an analysis that directly considers seams.)

An inspection of Figs. 10 and 11 shows that for a given geometry and seam distribution, the ratio $I^*/I_d$ in fact varies with load level (and hence $I_d$). For 20% (or less) seams along the socket, comparison of the results from a full seam analysis with the approximate relationship given by [15] indicates that for the short piers considered ($L/D = 2$) the approximate approach gives values of $I^*$ typically within 15% of the values deduced from a full finite element analysis. For long piers, the error associated with the approximate approach ([15]) can be somewhat greater (up to 25% for the very highly loaded piers...
considered); however, even for long piers, at typical load levels, the error associated with the approximate approach is less than 20%.

The value of $F^*$ to be used in association with [9], [13], and Figs. 6 and 7 can be estimated from Figs. 10 and 11 and additional charts given by Rowe and Armitage (1984); however, the results presented here suggest that for many practical situations it may be adequate to use the approximate approach ([15]) to determine $F^*$.

**Conclusion**

A theoretical approach for the prediction of the behaviour of piers socketed into soft rock has been described. This approach allows consideration of plastic failure within the rock, as well as slip, strain softening, and dilatancy at the pier-rock interface.

The fundamental difference between tests commonly used for determining average side shear resistance has been discussed. It was suggested that there will generally be a discrepancy between the average side shear obtained from displacement-controlled tests on side shear only sockets and that obtained from load-defined tests on end-bearing piers. This discrepancy is small for moderately long and rough sockets; however, the results obtained from side shear sockets may underestimate the available residual side shear resistance of end-bearing piers. The magnitude of this discrepancy will depend upon the relative stiffness of the pier and rock as well as the length and roughness of the socket. In theory, the discrepancy may be more than an order of magnitude for typical parameters and a perfectly smooth socket, although, in practice, this variation would not be expected because of the roughness of even relatively "smooth" sockets.

The effect of a number of fundamental parameters upon the average peak and residual side shear values was examined for end-bearing piers subjected to a load-defined test. It was shown that the magnitude of the average side shear depends not only upon the strength parameters, but also upon dilatancy at the interface, the length-to-diameter ratio, and the relative modulus of the pier and rock. The average side shear deduced using typical values of the fundamental parameters for two types of rock compared reasonably well with field values. These results suggest that the apparent scatter of results obtained from different field tests on rock with apparently the same strength characteristics may be partly due to the effect of parameters such as socket length, modulus ratio, and variable roughness of the socket walls.

A series of solutions as presented for a pier socketed into a homogeneous rock and for a pier end bearing on rock that is either stiffer or weaker than that along the shaft. In addition to the elastic response, these solutions provide a means of estimating the load at which full slip occurs and the corresponding pier head settlement, as well as the pier head displacement for any given load following full slip. Nonlinearity of the bearing strata may be approximated by the selection of an appropriate secant modulus.

Finally, the effect of weak horizontal seams along the pier shaft was examined and theoretical solutions for assessing the significance of these seams upon the pier response were presented. It is shown that an approximate approach that uses an equivalent modulus will give reasonable results for many situations where seams represent less than 20% of the socket length.

**Acknowledgements**

The research described in this paper was supported by the Department of Supply and Services Canada and the National Research Council of Canada under contract no. ISU83-00082.

Many thanks are due Dr. M. Bozozuk, of the National Research Council, who acted as Scientific Advisor on this project. Thanks are also due Mr. P. J. N. Pells for many useful discussions over many years.


Fig. 12. Design charts for a complete socketed pier \((E_p/E_r = 0.5; E_p/E_r = 10.0, 25.0, 50.0, \text{and} 100.0)\).


 Appendix

 Figures 12–14 provide design curves for values of \(E_p/E_r\) ranging from 10 to 250 for values of \(E_p/E_r\) of 0.5, 1.0, and 2.0. The use of these charts for the design of piers socketed into rock is described by Rowe and Armitage (1987a). However, it should be noted here that the effect of increasing pier length beyond that required to give a design settlement \(\rho_d\) depends on the relative stiffness of the concrete and rock. For modulus ratios \(E_p/E_r > 25\) increasing pier length will result in a decrease in settlement for a given load. However, in some cases involving stiffer rock (particularly when \(E_p/E_r \leq 10\)), increasing the length of the socketed pier may in fact increase settlement.
Fig. 13. Design charts for a complete socketed pier ($E_b/E_r = 1.0$; $E_p/E_r = 10.0, 25.0, 50.0, 100.0,$ and 250).
FIG. 14. Design charts for a complete socketed pier ($E_a/E_r = 2.0$; $E_p/E_r = 10.0, 25.0, 50.0,$ and $100.0$).

for high levels of load! This result arises from the fact that if the rock and concrete have comparable stiffnesses, and a bored pier is placed in the rock, then the interface between the rock and concrete represents a plane of weakness; the longer the pier, the more the rock is weakened and the larger will be the settlement once slip is induced at the interface. Clearly, if this is the case, then it is better to have a larger end-bearing pier at the surface of the rock (or perhaps socketed to a nominal $L/D = 1$) than to try and design a socketed pier.