The elastic displacements of single and multiple underream anchors in a Gibson soil

R. K. ROWE* and J. R. BOOKER†

An accurate and economical technique for the analysis of buried footings and multiple underream anchors in a non-homogeneous elastic soil is outlined. The method of analysis may be used for general non-homogeneous, anisotropic elastic soil profiles although, in this paper, attention is restricted to anchors in a soil whose modulus increased linearly with depth. The behaviour of both isolated underreams and multiple underream systems is examined. Consideration is given to the effect on anchor response of non-homogeneity, anchor embedment, layer depth and Poisson’s ratio as well as the number and spacing of underreams. The influence of anisotropy and anchor inclination are also discussed. The results from this study are presented in the form of influence charts which may be used as a hand method for estimating the elastic load-displacement behaviour for general anchor systems to sufficient accuracy for most practical purposes. The use of the charts is illustrated by two examples.

INTRODUCTION

The behaviour of anchor plates in an isotropic homogeneous elastic soil has been studied by a number of authors (Fox, 1948; Butterfield & Banerjee, 1971; Selvadurai, 1976; Rowe & Booker, 1979a, b). This research has provided elastic solutions which may be used to estimate the working load displacements of anchors in relatively homogeneous, isotropic soils for a wide range of geometric conditions. However, it is generally recognized that the assumptions of homogeneity and isotropy do not truly represent many soil deposits.

Soil deposits frequently exhibit appreciable increases in both elastic modulus and strength with depth, while horizontal modulus may commonly vary between one half and twice the vertical modulus depending upon the stress history of the deposit (Gerrard et al., 1972; Lo et al., 1977). Although engineers are aware of these characteristics of soil deposits, the availability of theoretical solutions for non-homogeneous soils, and the cost of obtaining detailed soil parameters are such that these soil profiles are often treated as being homogeneous and isotropic.

Consequently, the objectives of this paper are two-fold. Firstly, to provide a series of theoretical solutions for the behaviour of anchors in an idealized non-homogeneous soil profile; and secondly, to indicate the errors that would arise from the neglect of non-homogeneity and anisotropy.

Consideration will be given to the behaviour of single and multiple underream anchors with a vertical axis, resting in a non-homogeneous soil whose modulus increases linearly with depth as indicated in Fig. 1. For the purpose of analysis, it will be assumed that the anchor system may be idealized as consisting of rough, rigid, circular underreams which are fully bonded to the soil and are connected by perfectly smooth, rigid anchor rods of zero cross-section. The soil will be considered to be either isotropic, or cross-anisotropic with a plane of isotropy parallel to the soil surface. The soil mass is of infinite lateral extent and is either infinitely deep or else rests on a rough, rigid base at a depth D below the lowermost underream.

This non-homogeneous elastic profile could be analysed using finite element techniques; however a more efficient approach is to use a finite layer method which is outlined in this paper. This theory is implemented in the program FLANS (Finite Layer Analysis of Non-homogeneous Soils) which provides a convenient means of obtaining accurate solutions using relatively little input data, and is computationally more economical than the finite element method.
The results of this study are presented in the form of correction factors which may readily be used in hand calculations for estimating the effect of non-homogeneity or anisotropy upon anchor response, or for predicting the displacement of anchors in a non-homogeneous soil. The use of the correction factors is illustrated by a worked example in the Appendix.

It is considered that the solutions presented for single underream anchors could also be used to estimate the displacement of circular or near circular buried footings resting in a non-homogeneous soil. These solutions provide a more general alternative to the use of surface footing solutions in conjunction with depth correction charts such as those of Janbu, Bjerrum & Kjærnsli (1956) or Christian & Carrier (1978).

METHOD OF ANALYSIS

The behaviour of elastic, horizontally layered soil deposits can be analysed efficiently using finite layer techniques (Peutz, Van Kempen & Jones, 1968; Wardle & Fraser, 1975; Cheung, 1976). These methods are most suited for application to deposits which consist of several distinct layers and deposits which vary continuously must be approximated by a 'staircase' profile. Recently the Authors (Rowe & Booker, 1980b) have developed a method which is better suited to the treatment of a continuously varying deposit. In this method it is assumed that the deposit may be divided into a number of distinct layers \( z_i \leq z < z_{i+1} \) in which Poisson's ratio \( \nu_k \) is assumed to be constant and the elastic modulus is assumed to vary exponentially with depth, thus

\[
E = E_0 + \rho z
\]

where \( E_0 \) and \( \rho \) are constants. The modulus at depth \( z \) is given by the function

\[
E(z) = E_{m} \exp\left(-\frac{z}{z_m}\right)
\]

where \( E_{m} \) is the modulus at depth \( z = z_m \) and \( \mu_0 \) describes the variation of modulus with depth. An example of this approximation for material whose modulus varies linearly with depth is shown in Fig. 2.

The method, like the conventional finite layer approach, makes use of a Fourier transform to represent the displacements and stresses.

\[
(u_x, u_y, u_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (-iU_{xx}, -iU_{yy}, U_z) \exp\left[i(ax + by)ight] dx dy
\]

\[
\sigma_{xx}, \sigma_{yy}, \sigma_{zz} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (-iS_{xx}, -iS_{yy}, S_{zz}) \exp\left[i(ax + by)ight] dx dy
\]

Hooke's Law may be used to express the transformed stress components \( S_{xx}, S_{yy}, S_{zz} \) in terms of the transformed displacements \( U_{x}, U_{y}, U_{z} \).

\[
S_{xx} = \frac{\Omega_k}{1 + \nu_k} \exp[2\mu_0(z - z_0)] \left[ \frac{\partial U_x}{\partial z} - \alpha U_x \right]
\]

\[
S_{yy} = \frac{\Omega_k}{1 + \nu_k} \exp[2\mu_0(z - z_0)] \left[ \frac{\partial U_y}{\partial z} - \beta U_y \right]
\]

\[
S_{zz} = \frac{\Omega_k}{1 + 2\nu_k} \exp[2\mu_0(z - z_0)] \left[ \frac{\partial U_z}{\partial z} \right] + \left[ \alpha U_x + \beta U_y + (1 - \nu_k) \frac{\partial U_z}{\partial z} \right]
\]
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If equations (2) are substituted into the equilibrium equations it is found that the displacements may be expressed in the form

\[ U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \sum_{j=1}^{6} \xi_j \exp [p_j z] \]

where \( p_j \) and \( \xi_j \) are known (details of their calculation are given by Rowe & Booker, 1980b) and \( \xi_j \) are six constants to be determined.

The constants \( \xi_j \) may be calculated in terms of the 6 node plane displacements \( U_k, U_{k+1} \). Using these expressions equation (4) may be substituted into equation (3) to obtain the stiffness relation.

\[ \begin{bmatrix} T_k \\ T_{k+1} \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ B_k^T & C_k \end{bmatrix} \begin{bmatrix} U_k \\ U_{k+1} \end{bmatrix} \]

where \( T_k = +(S_{zz}, S_{zz}, S_{zz})^T \) is the traction on the node plane \( z = z_k \).

\( T_{k+1} = -(S_{zz}, S_{zz}, S_{zz})^T \) is the traction on the node plane \( z = z_{k+1} \).

\( U_{kk+1} = (U_k, U_k, U_k, U_{k+1}) \) are the displacements on the node planes \( z = z_{k+1} \).
and the matrices \( A_k, B_k, C_k \) are all known.

The conditions of displacement compatibility and traction equilibrium at the node planes may now be used to obtain the behaviour of a \( n \) layer system and it is found that

\[ \begin{bmatrix} A_1 & B_1 & 0 \\ B_1^T & C_1 + A_2 & B_2 \\ 0 & B_2 & C_2 + A_3 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ B_3 & \ldots & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_n \end{bmatrix} = \begin{bmatrix} C_1 + A_2 + A_3 \\ \vdots \\ \vdots \\ C_{n-2} + A_{n-1} + B_n \\ C_{n-1} + A_n \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_n \end{bmatrix} \]

where for simplicity it has been assumed that the layered system rests on a rough, rigid base and is subject to stress jump \( T_j \) at node plane \( j \).

Equation (6) may now be used to calculate the node plane displacements \( U_1, \ldots, U_n \) for any value of the parameters \( \alpha, \beta \) and so the actual displacements can be obtained by evaluating equation (2a) numerically. The stress components may be calculated similarly.

ANCHOR IDEALIZATION AND NUMERICAL DETAILS

The basic theory may be generalized to consider a number of different boundary conditions at the interface. However, attention will be restricted here to the displacement of anchor systems consisting of rough, rigid anchor plates which are fully bonded to soil which has been discussed by Rowe & Booker (1979a). Upon the response of anchors in a homogeneous soil which has been discussed by Row & Booker (1979a).

The overburden pressure gives rise to initial compressive stresses between the anchor plate and the soil. Loading the anchor decreases the compressive stress beneath the anchor until the stress becomes zero or tensile; at this point the anchor will break away from the underlying soil. The solutions for a fully bonded anchor presented in this Paper are valid until this breakaway occurs. Load path finite element results (up to collapse) indicate that under many circumstances the assumption of a fully bonded anchor is valid for working loads calculated by applying a factor of safety of 3 to the collapse load (see Rowe & Davis, 1980a, b).

The analysis of a rigid anchor is achieved by subdividing the anchor into a number of uniformly loaded ring "elements". The number of elements required to achieve 'rigidity' depends on the degree of non-homogeneity; however, it was found that the load-displacement relationship could be obtained to sufficient accuracy by subdivision of the anchor into four annular rings. Further subdivision of the anchor typically altered the displacement by less than \( 3\% \). Twelve subdivisions gave an accuracy of better than \( 1\% \) for problems where analytical solutions were known.

An alternative approach to the subdivision of the anchor into uniformly loaded elements is to consider the average displacement of an anchor which causes a vertical stress jump given by the expression

\[ \sigma_r(r) = \frac{1}{\pi B \sqrt{(B^2/2)^2 - r^2}} \begin{cases} 0 < r < B/2 \\ 0 \text{ elsewhere.} \end{cases} \]

This approach allows considerable computational savings and was found to give excellent results. For a homogeneous soil, this approach agrees exactly with the analytic solutions for the limiting cases of a surface footing and an infinitely deep anchor. For
Fig. 3. Effect of embedment and layer depth upon anchor displacement in a homogeneous soil; (a) $v = 0.3$; (b) $v = 0.5$
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intermediate embedments, the approach gives similar solutions to those obtained by subdivision of the footing into twelve elements and may be regarded as being more accurate than solutions obtained using only four subdivisions. The errors involved in the use of this stress jump are greatest for shallow anchors resting in a highly non-homogeneous incompressible soil, however even in these cases a comparison with the results obtained for anchors divided into uniformly loaded elements suggests that the approximate approach is accurate to within 2%. The majority of results reported in this Paper were obtained using this approximate approach. These solutions were checked against results obtained with up to twelve annular elements.

A second numerical approximation arises from the use of an exponential function to represent the linear increase in modulus with depth. The accuracy of this approximation depends upon the variation in modulus between the top and bottom of each layer. If the ratio is considered to be too large for a particular layer then the layers may be split into sublayers. The results presented in this Paper were obtained for a maximum ratio of a modulus between the top and bottom of each layer of 1:25. This ensures that the error in the representation of the soil modulus is always less than 0.4%. This is a very conservative numerical restriction but was adopted to ensure consistently accurate results. In practice one may perform analyses for modulus ratio of 1:5 without appreciably altering the results and much higher modulus ratios may be adopted without significant error for layers more than two footing widths (diameters) below the footing. For layers of infinite depth the tangential exponential approximation (see Fig. 2) was used at a distance of 20 anchor diameters below the lowest underream. Increasing this distance had no significant effect upon the anchor response.

The analysis was validated by comparison with published solutions and the results of the Author's finite element analyses. Good agreement was obtained with the published results for anchors in a homogeneous soil (Selvadurai, 1976; Rowe & Booker, 1980a) and for footings upon a soil with a linearly increasing modulus with depth (Brown & Gibson, 1979). Agreement to better than 1% was also obtained between this method and finite element calculation for a number of non-homogeneous soil profiles. The finite element results required considerably greater data preparation time and were computationally more expensive. The advantage of the Author's approach over finite elements increased rapidly with increasing number and spacing of underreams and layer depth.

SOIL PROFILE

The results presented in this Paper were obtained for an ideal, elastic soil mass with a Young's modulus in the vertical direction \( E_v \) at a depth \( z \) given by the expression

\[
E_v = E_0 + \rho z
\]

where

- \( E_0 \) is Young's modulus in the vertical direction at the soil surface;
- \( \rho \) is the rate of increase in vertical modulus with depth.

Unless otherwise stated, the soil may be assumed to be isotropic with variable Young's modulus \( E_v \) and a constant Poisson's ratio \( v \). For anisotropic deposits, the vertical modulus varied as indicated above while the ratios \( E_h/E_v \) and \( G_{vh}/E_v \) (where \( E_h \) is the horizontal Young's modulus and \( G_{vh} \) is the independent shear modulus) were considered constant throughout the deposit. The independent Poisson's ratio \( v_h \) (for the effect of horizontal stress on complimentary horizontal strain) and \( v_{vh} \) (for the effect of vertical stress on horizontal strain) were also constant throughout the stratum. The third Poisson's ratio \( v_{bh} \) (for the effect of horizontal stress on vertical strain) is not independent and is given by \( v_{bh} = v_{vb} E_h/E_v \).

RESPONSE OF A SINGLE UNDERREAM IN A NON-HOMOGENEOUS SOIL

Consider a single, rigid circular underream of diameter \( B \) resting at a depth \( h \) below the soil surface with a rough rigid base at a further distance \( D \) below the anchor as indicated in Fig. 3(a). The displacement \( \delta \) of this anchor may be written in the form

\[
\delta = P c_w M_{NB} R_N R_b / B E_*
\]

where

- \( P \) is the applied load;
- \( E_* \) is the vertical Young's modulus at depth \( h \) below the soil surface;
- \( c_w \) is the displacement factor for an anchor in an infinite isotropic homogeneous soil;
- \( M_{NB} \) is an influence factor for the effect of embedment and layer depth upon the displacement of an anchor in a homogeneous soil;
- \( R_N \) is a correction factor for the effect of non-homogeneity upon anchor displacement and is defined by the ratio \( R_N = \text{anchor displacement for the actual non-homogeneous deposit divided by anchor displacement for a homogeneous deposit with } E = E_* \);
- \( R_b \) is a correction factor for the effect of anisotropy upon anchor displacement.
and is defined by $R_e = $ anchor displacement for actual anisotropic deposit divided by anchor displacement for a similar isotropic deposit.

The displacement factor $c_{\infty}$ may be obtained from Selvadurai's (1976) analytical solution and for a circular disc is given by

$$c_{\infty} = \frac{(1 + \nu)(3 - 4\nu)}{8(1 - \nu)} \quad \text{(9a)}$$

where $\nu$ is Poisson's ratio for an isotropic soil. For an anisotropic soil it is convenient to define $c_{\infty}$ by setting $\nu = \nu_{sh}$ in equation (9a) (although it is recognized that this does not give the exact solution). The effect of anchor shape may be approximately considered by replacing equation (9a) by a similar expression which may be deduced from Selvadurai's solution for oblate and prolate spheroidal anchors. Square footings may be considered by replacing equation (9a) by the approximate expression

$$c_{\infty} \approx \sqrt{\frac{\pi(1 + \nu)(3 - 4\nu)}{16(1 - \nu)}} \quad \text{(9b)}$$

and defining $B$ to be the width of the footing.

The influence factor $M_{kD}$ which indicates the effect of embedment and layer depth upon anchor displacement for a homogeneous soil, is given in Fig. 3 for $\nu = 0.3$ and 0.5 respectively.

**Effect of non-homogeneity**

In the absence of elastic solutions for a non-homogeneous soil, a reasonable engineering approach to the prediction of anchor displacement would be to use elastic solutions for a homogeneous soil in conjunction with an appropriate value of Young's modulus $E_o$ determined at anchor level. This displacement would be given by the expression

$$\delta = P c_{\infty} M_{kD}/BE_o \quad \text{(10)}$$

Thus the correction factor $R_{\nu}$ in equation (8) indicates the error involved in this approximate approach. In the subsequent discussion, the effect of non-homogeneity upon anchor displacement will be considered in terms of a correction to the displacement predicted using equation (10). In equation (10), the use of the modulus at anchor level $E_o$ as opposed, say, to the modulus at the surface $E_s$ will have a significant effect upon predicted displacement. Equation (10) implicitly takes account of the general variation in modulus with depth and the correction factor $R_{\nu}$ to be discussed here takes account of the local variation in modulus above and below the anchor.

The correction factor $R_{\nu}$ is given in Fig. 4 for a number of cases. Here the non-homogeneity of the soil is expressed in terms of the dimensionless parameter $E_o/E_s$ which represents the ratio of the surface modulus divided by the modulus at anchor level. It will be recognized that the rate of increase in modulus with depth is a function of both the modulus ratio $E_o/E_s$ and the embedment ratio $h/B$.

The effect of non-homogeneity upon anchor displacement is greatest for shallow anchors in deep deposits of incompressible soil and least for deep anchors. The correction factor $R_{\nu}$ for shallow anchors varies appreciably with non-homogeneity and the displacement may be reduced by up to 20% even for relatively low levels of non-homogeneity (i.e. $E_o/E_s > 0.8$). Deep anchors are insensitive to local variations in modulus irrespective of the general rate of increase in modulus with depth.

The effect of non-homogeneity reflected by the parameter $R_{\nu}$ arises from the proximity of the soil surface and the underlying rigid base to the anchor. This is best illustrated by considering the variation in $R_{\nu}$ with embedment ratio $h/B$ and layer depth ratio $D/B$ as shown in Figs 5 and 6 respectively.
Fig. 5. Effect of embedment upon the non-homogeneity correction factor $R_N$: (a) $\nu = 0.3$; (b) $\nu = 0.5$
Embedment ratio has an appreciable effect upon the non-homogeneity correction for anchors at embedments less than five anchor diameters. For most practical purposes, the effect of non-homogeneity may be neglected for embedment ratios greater than 5 and these anchors may be regarded as being 'deep'. The effect of embedment is a function of both non-homogeneity and layer depth. For a given embedment ratio, the effect of non-homogeneity increases with increasing layer depth. The relative importance of layer depth is greatest for shallow anchors.

The foregoing discussion has considered the elastic response of anchors in a non-homogeneous soil as compared with their response in a homogeneous soil with modulus $E_a$. This approach indicates the errors involved in the use of homogeneous solutions for a soil which is truly non-
Table 1. Effect of homogeneity and embedment upon displacement: \( v = 0.3 \) \( D/B = \infty \)

<table>
<thead>
<tr>
<th>Non-homogeneity ratio ( \rho B/E_0 )</th>
<th>Maximum variation in dimensionless displacement ( \delta B/E_0 ) for ( 1 \leq h/B &lt; 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>38%</td>
</tr>
<tr>
<td>0.1</td>
<td>25%</td>
</tr>
<tr>
<td>1</td>
<td>5%</td>
</tr>
<tr>
<td>10</td>
<td>2%</td>
</tr>
</tbody>
</table>

homogeneous. Furthermore, Figs 3–6 may be used in conjunction with equation (8) to predict the displacement of anchors in a non-homogeneous soil for a wide range of geometries and soil profiles.

The effect of embedment, layer depth and non-homogeneity upon anchor response may also be considered in absolute terms rather than in terms of correction to homogeneous solutions. Clearly, increasing the depth to an anchor in a linearly non-homogeneous soil will result in a decreased displacement due to the effect of the higher soil modulus \( E_0 \) at the anchor level. This trivial effect may be excluded by considering the dimensionless displacement \( \delta B/E_0 \) of an anchor in a given soil where \( P, B \) and \( E_0 \) are specified and the depth \( h \) is varied.

A comparison of anchor response for different embedment ratios \( (h/B) \), layer depths \( (D/B) \) and non-homogeneity \( (\rho B/E_0) \) indicates that increasing non-homogeneity significantly decreases the importance of the embedment ratio. For example, the results shown in Table 1 indicate that embedment ratio has an appreciable effect upon anchor displacement in a homogeneous soil; however, for a highly non-homogeneous soil \( (\rho B/E_0 > 1) \), increasing the embedment from 1 to 20 has negligible effect upon the dimensionless displacement. This result arises from the fact that, for these highly non-homogeneous soils, the stiffness of the soil more than one anchor diameter above the anchor is very small compared with the stiffness below the anchor. This rapid change in stiffness with depth introduces a scale effect whereby there is an apparent anti-symmetry in the region of the anchor similar to that observed for a deep anchor. Consequently, for these highly non-homogeneous soils the embedded depth \( h \) has little effect provided \( h/B > 1 \).

A similar argument applies to the influence of a rough, rigid base below the anchor. Table 2 indicates the decrease in displacement of an anchor due to the presence of a rough, rigid base at a depth \( D \). The influence of the rigid base, which is greatest for an incompressible soil, decreases appreciably as the non-homogeneity of the soil \( (\rho B/E_0) \) increases, and may often be neglected for highly non-homogeneous soils.

Table 2. Effect of rough rigid base upon displacement

<table>
<thead>
<tr>
<th>Displacement for ( D/B = \infty )</th>
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<tr>
<td>( h )</td>
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<tr>
<td>------</td>
</tr>
<tr>
<td>1</td>
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<td>1</td>
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<td>20</td>
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<td>20</td>
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<tr>
<td>20</td>
</tr>
</tbody>
</table>

The solutions presented in Figs 3–6 were for Poisson’s ratio \( v = 0.3 \) and 0.5; however, the displacement for other values of Poisson’s ratio may be determined by interpolation using the results presented in Fig. 7. This figure shows the displacement \( \delta(v) \) of an anchor in a soil with a given value of Poisson’s ratio \( v \) divided by the displacement \( \delta(v = 0.5) \) of the same anchor in a soil with \( v = 0.5 \).

The placement of an anchor in a deep homogeneous deposit is relatively independent of Poisson’s ratio, typically varying by less than 11% over the entire range of values for \( v \). Even the displacement of a relatively shallow anchor \( (h/B = 1) \) is quite insensitive to Poisson’s ratio and indeed the displacement for \( v = 0 \) is very close to the value for \( v = 0.5 \). This is a marked contrast with the behaviour observed for surface footings where the displacement for \( v = 0 \) is 33% greater than that for \( v = 0.5 \). The transition between surface footing behaviour and deep anchor behaviour occurs very rapidly over embedment ratios ranging from \( h/B = 0 \) and \( h/B = 1 \); in fact most of the transition occurs between \( h/B = 0 \) and \( h/B = 0.5 \). These results are of some practical significance since they suggest that the consolidation settlement of a buried footing will be considerably less than that for a surface footing.

The importance of Poisson’s ratio increases as the layer depth is reduced although it is still appreciably less than that observed for surface
Non-homogeneity of the soil significantly increases the importance of Poisson’s ratio. This is particularly true for shallow anchors in deep highly non-homogeneous deposits. The effect of non-homogeneity upon the sensitivity of displacements to the value of Poisson’s ratio is reduced for deep anchors or shallow layers. The variation in displacement with Poisson’s ratio for a non-homogeneous soil may be considered to be the same as that for a homogeneous soil for anchors buried to a depth $h$ greater than the depth $D$ of soil beneath the anchor ($h/D > 1$). In the case of deep layers, this approximation may be used for $h/B > 20$.

Effect of anisotropy

The depositional and stress histories of many soil deposits are such that over an appropriate stress range, they may be idealized as a cross-anisotropic elastic medium. Although this situation is well recognized, the difficulties and expense of obtaining anisotropic soil parameters are such that foundation engineers frequently treat these soils as being isotropic with a modulus $E$ (equal to $E_a$) and Poisson’s ratio $v$ (typically equal to $v_{eh}$) where these parameters may be determined directly from a simple triaxial test. In view of this situation, it is of some practical interest to estimate the errors in displacement predictions that would arise from the use of the isotropic elastic solutions presented in the previous section in conjunction with the isotropic parameters $E = E_a$ and $v = v_{eh}$ for a soil which is truly anisotropic.

Analyses were performed for a number of anisotropic soil profiles, geometries and degrees of non-homogeneity. Details regarding the anisotropic parameters examined are given in Table 3. These parameters are considered to cover the range of anisotropy that might be expected in many soil deposits and were selected with regard to the available experimental data relating to drained anisotropic soil parameters.

The results from these analyses were expressed in terms of an anisotropy correction factor $R$ which represents the ratio of the displacement of an anchor in the anisotropic soil divided by the displacement of a similar anchor in an isotropic soil with $E = E_a$ and $v = v_{eh}$.

### Table 3. Anisotropic soil parameters

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$E_a/E_e$</th>
<th>$v_{eh}$</th>
<th>$v_{eb}$</th>
<th>$v_{eb}/E_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.35</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>0.1</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>0.1</td>
<td>0.6</td>
<td>0.35</td>
</tr>
<tr>
<td>d</td>
<td>2</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Fig. 7. Variation in displacement with Poisson’s ratio; (a) $h/B = 1$; (b) $h/B = 3$; (c) $h/B = 20$
Table 4. Variation in displacement with anisotropy for a highly non-homogeneous soil; $E_0/E_a = 0.02$

<table>
<thead>
<tr>
<th>h/B</th>
<th>$D$</th>
<th>$R_s$</th>
<th>Anisotropy ratio</th>
<th>Displacement (anisotropic)</th>
<th>Displacement (isotropic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1.147</td>
<td>1.133</td>
<td>0.941</td>
<td>0.698</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>1.156</td>
<td>1.14</td>
<td>0.940</td>
<td>0.691</td>
</tr>
<tr>
<td>1</td>
<td>$\infty$</td>
<td>1.146</td>
<td>1.098</td>
<td>0.960</td>
<td>0.682</td>
</tr>
</tbody>
</table>

The effect of anisotropy is relatively insensitive to the non-homogeneity of the soil profile. Thus the anisotropy correction factor $R_s$ varied by less than 5% and typically less than 2% for soil profiles ranging from homogeneous to highly non-homogeneous ($E_0/E_a = 0.02$). Similarly, the effect of anisotropy was relatively independent of anchor embedment and layer depth with variations of less than 5% over the entire range of geometries studied ($h/B \geq 1$, $D/B \geq 3$). A number of typical results are given in Table 4.

Relative to the isotropic solutions, the displacement of an anchor increased for $E_h < E_a$ and decreased for $E_h > E_a$. For values of the independent shear modulus $G_h$, which were close to the isotropic value, the effect of anisotropy was modest, varying by less than 1% for values of $E_h/E_a$ in the range from 0.5 to 2. However, the independent shear modulus did vary an appreciable effect upon the anchor displacement, and it may be regarded as being a particularly important anisotropic parameter.

The independent shear modulus is frequently deduced from values of Young's modulus determined for samples cut at different orientations to the vertical. The fact that this quantity is derived from the results of a number of other tests suggests that it is likely to be the least accurately determined parameter. In view of the above finding regarding the significance of $G_h$ as a parameter, it would seem to be more appropriate to determine this parameter directly rather than by deduction.

The results of this investigation into the effect of anisotropy upon anchor displacement suggest that for many soils the neglect of anisotropy would lead to errors of less than 15% and in many cases, anisotropy may be neglected. The results given in Table 4 may be used as a guide to the effect of anisotropy upon anchor displacement for soils with parameters similar to those given in Table 3. The insensitivity of the correction factor $R_s$ to anchor depth, layer depth and non-homogeneity also suggests that the ratio $R_s$ for an anchor will be similar to the value determined for a surface footing on a deep homogeneous deposit. This value may be deduced from the results published by Gerrard & Harrison (1970). Checks indicate that this approximate approach would generally give values of $R_s$ accurate to within 10%. Clearly, if anisotropy is considered to be important, then a full analysis may be performed as described previously.

THE DISPLACEMENT OF MULTIPLE UNDERREAM ANCHORS IN A NON-HOMOGENEOUS SOIL

The results presented for single underream anchors may be used to obtain an estimate of multiple underream anchor behaviour. Assuming for the moment that there is no interaction between underreams, then the displacement $\delta_i$ of any underream $i$ at a depth $h_i$ and underlain by soil to further depth $D_i$ is given by equation (8)

$$\delta_i = \frac{P_i E_a}{BE_a (E_h/E_a)}$$

(11)

where

- $P_i$ is the load causing a displacement $\delta_i$;
- $E_a$ is the modulus at the level of the top underream;
- $E_h$ is the modulus at depth $h_i$;
- $M_i = M_{AD} R_s R_a$ is determined for the appropriate geometric parameters $h_i/B$, $D_i/B$ using the results of the previous section.

Considering a multiple underream anchor system with $n$ underreams connected by inextensible anchor rods, the displacement of each underream will have the same value $\delta_0$. (Generalization of this approach to the case involving extensible anchor rods is straightforward.) The total load $P$ required to cause this displacement $\delta_0$ of the anchor system would be

$$P = \frac{B \delta_0 E_a}{E_h} \sum_{i=1}^{n} \frac{E_h/E_a}{M_i}$$

or conversely the displacement due to the applied load $P$ would be

$$\delta = \frac{P E_h}{BE_a} \left( \sum_{i=1}^{n} \frac{E_h/E_a}{M_i} \right)^{-1}$$

This procedure takes account of the effect of non-homogeneity but will generally underestimate the settlement since the effect of interaction has been
Fig. 8. Interaction factor; (a) for $h/B = 1$, $D/B = \infty$, $v = 0.5$; (b) for $h/B = 10$, $D/B = \infty$, $v = 0.5$
ELASTIC DISPLACEMENTS OF UNDERREAM ANCHORS IN A GIBSON SOIL

Fig. 9. Effect of embedment ratio upon interaction between underreams; (a) 2 underreams; (b) 5 underreams

Fig. 10. Effect of non-homogeneity upon interaction between underreams; (a) 2 underreams; (b) 5 underreams

neglected. The true displacement of the underream system will be given by

$$\delta = \frac{Pc_{m}}{BE_{a}} \left( \sum_{i=1}^{n} \frac{E_{i}/E_{a}}{M_{i}} \right)^{-1} R_{f}$$

where $R_{f}$ is an interaction factor indicating the increase in displacement of the anchor system due to interaction between the underreams.

The interaction between underreams is primarily a function of the number of anchor spacing and spacing of underreams, although the level of interaction is affected by the non-homogeneity of the soil and the proximity of the anchor to the free surface or a rigid base. The interaction factor $R_{f}$ is given in Fig. 8 for a number of anchor systems involving between two and five underreams, equally spaced at a distance $s$ apart. Interaction effects are seen to be quite significant. The results show that interaction effects are greatest for a large number of closely spaced anchors at shallow depth in a deep homogeneous deposit. For two underream anchors, the effect of interaction may be neglected (to an accuracy of better than 10%) for spacings ranging from $3B$ to $10B$ depending upon the non-homogeneity of the soil. However for five underreams, the effect of interaction can only be neglected at large spacings ranging from 12 to 20. For common underream systems, the effect of interaction upon displacement is likely to be between 10% and 100%.

From the results given in Fig. 8, it is clear that the proximity of the free surface has some effect upon interaction between anchors. This aspect of anchor
Predicted displacement using homogeneous solutions
Actual displacement for the non-homogeneous profile

<table>
<thead>
<tr>
<th>$h/B$</th>
<th>$E_0/E_a$</th>
<th>$pB/E_0$</th>
<th>$S/B = 1$</th>
<th>$S/B = 5$</th>
<th>$S/B = 1$</th>
<th>$S/B = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>49</td>
<td>1.64</td>
<td>1.45</td>
<td>1.56</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>3</td>
<td>1.57</td>
<td>1.43</td>
<td>1.53</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1</td>
<td>1.47</td>
<td>1.38</td>
<td>1.47</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.43</td>
<td>1.36</td>
<td>1.33</td>
<td>1.39</td>
<td>1.29</td>
</tr>
<tr>
<td>10</td>
<td>0.02</td>
<td>49</td>
<td>1.67</td>
<td>1.10</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.3</td>
<td>1.07</td>
<td>1.09</td>
<td>1.10</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.1</td>
<td>1.06</td>
<td>1.07</td>
<td>1.08</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.043</td>
<td>1.04</td>
<td>1.06</td>
<td>1.06</td>
<td>1.07</td>
</tr>
</tbody>
</table>

The effect of non-homogeneity upon interaction factor $R_f$ is shown in Fig. 10. For a shallow anchor, the interaction decreases significantly with increasing non-homogeneity. Most of the effect of non-homogeneity occurs for moderately non-homogeneous soils where $E_0/E_a$ lies in the range 0.5–1. Interaction between the underreams of a deep anchor system is relatively insensitive to the level of non-homogeneity.

Figures 8 and 10 provide sufficient detail to allow the determination of $R_f$ by interpolation where necessary, for a wide range of soil profiles and anchor geometries.

In the absence of details regarding the variation in elastic modulus with depth, an approximate engineering approach (for preliminary design purposes) would be to use elastic solutions for a homogeneous soil (Rowe & Booker, 1980a) in conjunction with an elastic modulus determined at a representative depth. A reasonably representative depth might be considered to correspond to the centre of the proposed anchor system. The error associated with this approximation may be assessed by comparing the predicted displacement with the actual displacement calculated for the non-homogeneous profile, as indicated for a number of cases in Table 5. The approximate approach considerably overestimates the displacement for shallow anchor even for moderately non-homogeneous soils. This situation arises because of the relative insensitivity of anchors to embedment depth for a non-homogeneous soil, as discussed in the previous section. The approximate approach provides quite reasonable estimates of anchor displacement for deep anchors and, as was the case for isolated underreams, the effect of non-homogeneity may be largely neglected for deep anchors provided a representative value of elastic modulus (corresponding to the centre of the anchor systems) is used in conjunction with the homogeneous elastic solutions.

### Effect of Poisson’s ratio

The interaction factor $R_f$ given in Figs 8–10 was determined for an incompressible soil with $v = 0.5$. This value of Poisson's ratio corresponds to the worst case and the interaction between underreams in a compressible soil ($v < 0.5$) could be conservatively predicted using equation 12 by determining $M_f$ for the appropriate value of Poisson's ratio $v$ and then using this in conjunction with the interaction factor $R_f$ given in Figs 8–10 for $v = 0.5$. To indicate the likely error in this approach, the effect of Poisson’s ratio upon the interaction factor $R_f$ is shown in Fig. 11 for a number of cases.

The effect of Poisson’s ratio upon interaction between underreams is greatest for anchors with a large number of closely spaced underreams, and is completely negligible for underreams at large spacings. For the cases considered in Fig. 11, the effect of Poisson's ratio upon $R_f$ is always less than 20% and is frequently less than 10%. For most practical cases, the effect of Poisson’s ratio upon $R_f$ could be neglected, although the results presented in Fig. 11 do provide a convenient means of taking this effect into consideration.

### Effect of layer depth

The values for the interaction factor $R_f$ given in Figs 8–11 are for a soil deposit of infinite depth. These values may be conservatively used for layers...
ELASTIC DISPLACEMENTS OF UNDERREAM ANCHORS IN A GIBSON SOIL

The effect of anisotropy upon the interaction between underreams is greatest for five closely spaced \((S/B = 1)\) underreams in a soil type \(d\) \((E_d/E_s = 2, G_d/E_s = 0.8)\). For this case the interaction is reduced. For soils with \(E_d/E_s < 1\), interaction between underreams was increased by up to 5%.

On the basis of these results for the anisotropic soils indicated in Table 3, it would appear that the effect of anisotropy may be conveniently included in the displacement prediction by determining the displacement for the anchor system in an isotropic soil and then multiplying this by a typical (or average) value of \(R_s\) determined for the individual underreams as described in the previous section.

INCLINED ANCHORS

The method of analysis and results presented in this Paper are for anchor systems with a vertical axis. However, on the basis of these results and the earlier work by Rowe & Booker (1979b, 1980) a number of tentative suggestions can be made regarding the estimation of anchor displacement for systems with an axis inclined at some angle to the vertical.

It is suggested that the displacement of a single underream anchor with an axis at 60° or less to the vertical may be determined by calculating the displacement for a homogeneous soil (Rowe & Booker, 1979b) with a modulus \(E_a\) equal to the modulus of the centroid of the anchor. This displacement should then be multiplied by a correction factor \(R_n\) determined from the results given in this Paper for an embedment ratio \(h/B\), where \(h\) is the depth to the centroid of the anchor.

Published results for multiple anchors in a homogeneous soil (Rowe & Booker, 1980a) indicate that anchor response is varied by less than 10% for anchors inclined at up to 75° from the vertical. In view of this finding, combined with the relative insensitivity of the interaction factor \(R_s\) to the degree of non-homogeneity, it is suggested that the displacement of an anchor system with an inclined axis could be estimated using equation 12, where the single underream terms \(M_j\) are determined as described above and where the interaction factor \(R_s\) is selected for the true spacing \(S/B\), and an embedment ratio \(h/B\) corresponding to the vertical distance from the centroid of the leading anchor to the soil surface.

CONCLUSIONS

An accurate and economical technique for analyzing multiple underream anchors in a non-
homogeneous soil has been outlined. The method of analysis may be used for quite general non-homogeneous, anisotropic elastic soil profiles although in this paper attention is restricted to anchors in a soil whose modulus increases linearly with depth.

The behaviour of an isolated underream (or buried footing) was examined. Consideration was given to the depth of embedment, layer depth, and the degree of non-homogeneity. From these results, the following was concluded.

(a) The effect of non-homogeneity is greatest for shallow anchors in deep deposits, and least for deep anchors.
(b) Anchor response is highly dependent upon embedment ratio for anchors with an embedment ratio less that 5 in a soil with relatively low non-homogeneity.
(c) For most practical purposes, an anchor with an embedment ratio greater than 5 may be regarded as being deep and the proximity to the soil surface may be neglected.
(d) Anchors in highly non-homogeneous soils are insensitive to the proximity of a free surface for embedment ratios greater than 1.
(e) A rough, rigid base less than ten diameters below an anchor has an appreciable effect upon anchor response for soil of relatively low homogeneity. Layer depth is a secondary parameter for highly non-homogeneous soils.
(f) Poisson's ratio has a small effect upon anchor displacement for all but very shallow anchors and it is suggested that the consolidation settlement of buried footings will be considerably less than that for surface footings.
(g) Anisotropy has relatively little effect upon anchor response for many typical soil profiles where the independent shear modulus is not appreciably different from the isotropic value. The independent shear modulus is an important parameter and may significantly influence the anchor response. It is suggested that, where possible, this parameter should be measured directly rather than deduced from other quantities as is common practice.

The displacement of multiple underream anchors may be predicted using the results for single underreams in conjunction with a series of interaction factors. From the study of multiple underream behaviour, the following is concluded.

(a) For common underream systems, the effect of interaction upon displacement is likely to be between 10% and 100%.
(b) The effect of proximity to the soil surface upon underream interaction may be neglected for embedment ratios greater than 5. Interaction between underreams for these deep anchors is relatively insensitive to the level of non-homogeneity.
(c) Interaction is greatest for anchors in an incompressible soil; however the influence of Poisson's ratio upon interaction is modest for closely spaced underreams, and may be neglected at large spacings.

Parametric solutions are presented in the form of influence charts which may be used directly in hand calculations to predict the elastic load-deflexion behaviour of single and multiple underream anchors for a wide range of parameters.

ACKNOWLEDGEMENT

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REFERENCES

Fox, E. N. (1948). The mean elastic settlement of a uniformly loaded area at depth below the ground surface. Proc. 2nd Int. Conf. Soil Mech., Rotterdam 1, 129–132.
APPENDIX 1

ILLUSTRATIVE EXAMPLE

To illustrate the use of the charts presented in this paper for estimating the elastic displacement of anchor systems, consideration will be given to two examples.

EXAMPLE 1

Estimate the displacement of a single footing (anchor) 1 m in diameter when subjected to a load of 1 MN. The footing is buried to a depth of 1.5 m in a sand deposit of total depth 16.5 m. The sand is considered to have a modulus which increases linearly with depth. The modulus is zero at the surface and 13 MPa at 1.5 m. Assume \( \nu = 0.2 \).

Calculation

From the above data, the relevant dimensionless parameters may be determined

\[
D/B = (16.5 - 1.5)/1 = 15
\]

\[
h/B = 1.5
\]

Table A1. Determination of \( R_N \) by interpolation

<table>
<thead>
<tr>
<th>Step</th>
<th>Comment</th>
<th>( h/B )</th>
<th>( D/B )</th>
<th>( E_0/E_1 )</th>
<th>( R_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Target case</td>
<td>1.5</td>
<td>15</td>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>Select case closest to target in Fig. 5: basic case</td>
<td>1.5</td>
<td>( \infty )</td>
<td>0.045</td>
<td>0.78</td>
</tr>
<tr>
<td>2a</td>
<td>Examine the effects of ( E_0/E_1 ) from Fig. 4 determine ( R_N ) for two cases</td>
<td>1</td>
<td>( \infty )</td>
<td>0</td>
<td>0.695</td>
</tr>
<tr>
<td>2b</td>
<td>Ratio of these gives the variation in displacement due to ( E_0/E_1 ); therefore improved estimate of ( R_N = 0.78 \times 0.695 = 0.777 )</td>
<td>1.5</td>
<td>( \infty )</td>
<td>0</td>
<td>0.777</td>
</tr>
<tr>
<td>3a</td>
<td>Examine the effect of ( D/B ); From Fig. 6 determine ( R_N ) for two cases</td>
<td>1</td>
<td>15</td>
<td>0.045</td>
<td>0.725</td>
</tr>
<tr>
<td>3b</td>
<td>Ratio of these results gives the variation in displacement due to ( D/B ); therefore improved estimate of ( R_N = 0.777 \times 0.725 = 0.7807 )</td>
<td>1.5</td>
<td>15</td>
<td>0</td>
<td>0.7807</td>
</tr>
</tbody>
</table>

Since solutions are given for \( \nu = 0.3 \) and 0.5 only, select the charts closest to the relevant value of \( \nu \). Here take \( \nu = 0.3 \). Therefore from equation (9)

\[
c_\nu = \frac{(1 + \nu)(3 - 4\nu)}{8(1 - \nu)} = 0.418
\]

From Figure 3(a), \( h/B = 1.5 \) and interpolating between \( D/B = 10 \) and 20 for the actual value of \( D/B = 15 \), \( M_{00} = 1.2 \). The non-homogeneity correction \( R_N \) may now be determined by interpolating from Figs 4-6, as shown in Table A1, and is found to be 0.807. Neglecting anisotropy, the displacement of the footing can now be determined for \( \nu = 0.3 \). Therefore

\[
\delta = \frac{P_c \cdot M_{00} \cdot R_N \cdot R_s}{B \cdot E_s}
\]

\[
= \frac{1}{1 \times 13} \times 0.418 \times 1.2 \times 0.807
\]

\[
= 0.031 \text{ m}
\]

The displacement for \( \nu = 0.2 \) may be estimated from Fig. 7 by interpolation for \( D/B = \infty \), \( E_0/E_1 = 0.045 \) (Table A2). Therefore for \( \nu = 0.2 \), \( \delta = 0.031 \times 0.019 = 0.0032 \text{ m} \). (The effect of Poisson's ratio is clearly insignificant for this case.) This displacement was obtained by interpolation from the charts given in the paper and is within 1% of the value obtained from a direct analysis.

EXAMPLE 2

Estimate the displacement of a multiple anchor which may be idealized as a multiple underream system consisting of three underreams of 0.5 m dia. at depths of 0.5 m, 1.5 m and 2.5 m. The anchor rests in a deep clay deposit with a surface modulus of 3 MPa and a rate of increase in modulus with depth of 2 MPa/m. The anchor is subjected
to a load of 1 MN. Assume $\nu = 0.3$.

**Calculation**

From the above data, the relevant dimensionless parameters are

$$
\frac{D}{B} = \infty \quad \frac{h}{B} = 1 \quad \frac{S}{B} = 2
$$

no. of underreams $m = 3$

From this, $E_i = E_o + \rho h = 3 + 2 \times 0.5 = 4$ MPa, and therefore $E_o/E_i = 0.75$. The displacement of the anchor system is given by equation (6).

$$
\delta = \frac{P c x \left( \sum_{i=1}^{3} \frac{E_i/E_o}{M_i} \right)^{-1}}{B E_o} R_i
$$

For $\nu = 0.3$, $c_x = (1 + \nu)(3 - 4\nu)/(8(1 - \nu)) = 0.418$. The value of $M_i = M_{sys}$, $R_i$ for each anchor may be determined as described in the previous example; this is illustrated in Table A3. From Table A3

<table>
<thead>
<tr>
<th>Underream</th>
<th>$h/B$</th>
<th>$D_i/B$</th>
<th>$E_i$</th>
<th>$E_o/E_i$</th>
<th>$M_{sys}$</th>
<th>$R_i$</th>
<th>$M_i$</th>
<th>$E_o/E_a$</th>
<th>$M_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>0.75</td>
<td>1.4 \quad 0.875 \quad 1.148</td>
<td>1.045</td>
<td>1.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>0.5</td>
<td>1.14 \quad 0.9 \quad 0.926</td>
<td>1.025</td>
<td>1.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>8</td>
<td>0.375</td>
<td>1.085</td>
<td>0.93 \quad 1.009</td>
<td>2</td>
<td>0.9371</td>
<td>4.32</td>
<td></td>
</tr>
</tbody>
</table>

This displacement which was calculated by interpolation using the chart given in the Paper may be compared with the displacement of 0.067 m determined from a direct analysis.

<table>
<thead>
<tr>
<th>$h$</th>
<th>$D$</th>
<th>$E_0$</th>
<th>$S$</th>
<th>$m$</th>
<th>$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\infty$</td>
<td>0.75</td>
<td>2</td>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>$\infty$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1.66</td>
</tr>
<tr>
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<td>0.75</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1.39</td>
</tr>
<tr>
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<td>0.75</td>
<td>2</td>
<td>3</td>
<td>1.52</td>
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</table>