EVALUATION OF TWO ANALYTICAL EQUATIONS FOR LEAKAGE THROUGH HOLES IN COMPOSITE LINERS

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ABSTRACT
The prediction of leakage through holes in composite liners obtained using two analytical equations are compared with results from finite element analysis. One equation examines leakage though circular holes in a geomembrane (GM) and the second equation can be used to estimate leakage through holed wrinkles in a GM. Composite liners involving both geosynthetic clay liners (GCL) and compacted clay liners (CCL) (i.e., GM/CCL and/or GM/GCL combinations) are examined. It is shown that for the range of cases examined, the analytical equations provide predictions that are sufficiently accurate for practical purposes.

1 INTRODUCTION
Composite liners involving a geomembrane (GM) over a geosynthetic clay liner (GCL) or compacted clay liner (CCL) are widely used to control contaminant escape from containment facilities such as landfills. The presence of holes in the GM provides an opportunity for leakage of leachate through the composite liners. Holes may occur in a GM in direct contact with the underlying clay liner or in wrinkles that form in the GM (Chappel et al., 2007). Rowe (1998) presented analytical equations that could be used to assess leakage for both cases. These equations involve some approximation and hence the question arises as to how they compare with finite element results that involve fewer assumptions. Thus the objective of this paper is to compare the predicted flow (leakage) obtained using Rowe’s (1998) equations with those calculated using finite element analysis.

2 FOOSE ET AL. (2001) STUDY
Foose et al. (2001) used a three-dimension finite difference model (MODFLOW) to analyze leakage through circular holes in flat GM for composite liners involving GCLs. The 3-D numerical results showed good agreement with the analytical solution of Rowe (1998) over the practical range of interface transmissivity identified by Harpur et al. (1993). This paper extends the range of cases examined by Foose et al. (2001) and in addition examines leakage through holed wrinkles.

3 ANALYTICAL EQUATIONS
Rowe (1998) provided a solution for leakage, \( Q \) (\( m^3/s \)), through a single circular hole in a GM

\[
Q = \alpha k \left[ r_0^2 i_s + 2 i_s \Delta_1 + 2 \Delta_2 - \frac{2h_w}{H_L + H_f} \Delta_2 \right] \tag{1a}
\]

where

\[
\Delta_1 = \left[ r_w A_1 (r_w, r_w) K_1 (\alpha w) + r_w A_2 (r_w, r_w) J_1 (\alpha w) \right] + \frac{r_0 A_1 (r_0, r_w) K_1 (\alpha 0)}{\alpha} \frac{r_0 A_2 (r_0, r_w) J_1 (\alpha 0)}{\alpha} \tag{1b}
\]

and

\[
\Delta_2 = \left[ -r_w A_2 (r_w, r_w) K_1 (\alpha w) - r_w A_2 (r_w, r_0) J_1 (\alpha w) \right] + \frac{r_0 A_2 (r_0, r_0) K_1 (\alpha 0)}{\alpha} \frac{r_0 A_2 (r_0, r_w) J_1 (\alpha 0)}{\alpha} \tag{1c}
\]
and \( h_w \) is the height of leachate above the liner (m), \( k_o \) the harmonic mean hydraulic conductivity of the liner and foundation layer (ms\(^{-1}\)), \( H_o \), \( H_f \) the thickness of the liner and attenuation layer above an underlying aquifer, respectively (m), \( r_o \), the radius of the single hole (m), and \( r_w \) is the wetted radius (which can be calculated as described by Rowe, 1998), and \( K_o \), \( I_o \), \( K_f \) and \( I_f \) are Bessel functions that can be evaluated from functions as per Press et al (1986). This equation has been implemented as part of a commonly used contaminant transport model POLLUTE V7™ (Rowe and Booker, 2005).

Rowe (1998) also provided an analytical equation for the rate of leakage through a hole in a wrinkle in the GM (referred as a holed wrinkle) where the hole is of sufficient size not to control leakage (i.e. as defined by Bernoulli’s equation). This leakage, \( Q \) (m\(^3\)/s) per holed wrinkle of length \( L \) (m) is given by:

\[
Q = 2Lk_o \left[ b + \frac{1}{\alpha} \left( e^{-\alpha (b-d)} - 1 \right) \right] \left( \frac{H_L + H_f + h_w - h_o}{H_L + H_f} \right)
\]  

(2a)

where

\[
\alpha = \sqrt{\frac{k_s}{H_L + H_f}} \theta_k
\]  

(2b)

\[
\frac{H_L + H_f}{k_s} = \frac{H_L}{k_L} + \frac{H_f}{k_f}
\]  

(2c)

\[
C = H_L + H_f - h_o
\]  

(2d)

and \( h_o \) is the height of leachate above the liner (m), \( h_w \) the hydraulic head at the bottom of the attenuation layer (m), \( k_o \) the harmonic mean hydraulic conductivity of the liner and foundation layer (ms\(^{-1}\)), \( H_o \), \( H_f \) the thickness of the liner and attenuation layer above an underlying aquifer, respectively (m), \( k_s \), \( k_f \) the hydraulic conductivity of the liner and attenuation layer, respectively (ms\(^{-1}\)), \( \theta_k \) the transmissivity of the interface between the GM and the clay liner (m\(^2\)/s), \( w \) the width of the GM wrinkle (m), and \( H \) is the spacing between parallel wrinkles (m).

Once leakage is calculated from a single hole or holed wrinkle, it can be used to calculate the leakage per hectare by multiplying the value of \( Q \) obtained from Eq. 1 or Eq. 2 by the number of holes or holed wrinkles per hectare. Typically this leakage is converted from m\(^3\)/s ha\(^{-1}\) units to litres per hectare per day (lphd).

4 MODELLING

The Finite Element program SEEP/W v5® was used to model both axi-symmetric conditions for a circular hole in a GM in direct contact with the clay liner. Holed wrinkles were modelled as a liner features assuming a 2D cross-section. Two different cases were modeled. Case 1 involved circular holes in a flat GM. Case 2 considered a series of holed wrinkles on a flat base. In the two cases the GM was modeled as a no-flow boundary except at the location where a wrinkle or hole exists. At the location of the holes or wrinkle, a typical design leachate head, \( h_w \), of 0.3m was specified.

The interface between the GM and the clay liner was modeled with a transmissivity, \( \theta \), in the range from \( 2 \times 10^{-12} \) to \( 2 \times 10^{-10} \) m\(^2\)/s for the GCL based on Harpur et al. (1993) and from \( 1.6 \times 10^{-8} \) to \( 1 \times 10^{-7} \) m\(^2\)/s for CCL based on “good” and “poor” interface conditions as defined by Giroud and Bonaparte (1989a-b). The clay liner thickness, \( H_L \), was taken to be 0.01 m for the GCL, and was underlain by 0.5 m attenuation layer (\( k_f = 1 \times 10^{-6} \) m/s). The thickness of the CCL was taken to be 0.5 and 0.9 m. The hydraulic conductivity \( k_o \) of the GCL examined ranged between \( 2 \times 10^{-12} \) and \( 2 \times 10^{-10} \) m/s to represent, the low end, a typical low value for CCL consolidated under the weight of overlying waste and, at the high end, the value one may get if there were significant clay-leachate interaction at the upper end (based on Petrov and Rowe 1997). The hydraulic conductivity of the CCL was taken to be between the \( 5 \times 10^{-3} \) m/s and \( 1 \times 10^{-10} \) m/s.

A series of numerical experiments with a variety of mesh configurations was used to establish a suitable numerical scheme. Based on this study, the barrier systems examined in this study were modeled with between 12000 and 52300 linear triangular elements. Steady state and saturated conditions were examined.

5 RESULTS AND DISCUSSION

5.1 Leakage through a Circular Hole (Case 1)

Analyses were performed for 1 cm radius circular hole in a GM in direct contact with the underlying GCL or CCL for the range of parameter combinations described in the previous section. The results for the analytical equation were obtained from POLLUTEV7™ and the finite element results from SEEP/W® in an axi-symmetric mode. Figure 1 shows the leakage rate through hole in a GM over a GCL with hydraulic conductivity of \( 5 \times 10^{-11} \) m/s, calculated by the two methods over the practical range of interface transmissivity reported by Harpur et al. (1993) and a range of holes per hectare of 2.5 to 12 typically reported in the literature (see Rowe 2005 for a discussion).

As can be seen, there is excellent agreement between the results with the discrepancy of less than 4% over the range of interface transmissivity examined being of no practical significance.
Figure 1 Comparison of leakage through 1 cm radius hole for 2.5 holes/ha and 12 holes/ha calculated using Eq. 1 and FEM for different GM-GCL interface transmissivities. GCL hydraulic conductivity of 5x10^{-11} m/s.

Figure 2 examines the effect of the hydraulic conductivity of the GCL on the calculated leakage for an interface transmissivity of 2 x 10^{-10} m²/s. Again it can be seen that there is excellent agreement between the analytical solution and the finite element method over the practical range of hydraulic conductivity between 2 x 10^{-12} and 2 x 10^{-10} m/s with a maximum discrepancy of 0.3%.

A comparison of the results in Figures 1 and 2 shows that over the typical range of transmissivity and hydraulic conductivity, the transmissivity has a much greater effect on the leakage through a hole in a GM over a GCL. In all cases, very low leakage values are calculated (less than 1 lphd and mostly less than 0.1 lphd). This highlights the effectiveness of composite liners involving a GCL even if there was sufficient clay-leachate interaction to increase the GCL hydraulic conductivity to 2 x 10^{-10} m/s.

Figure 3 shows the leakage calculated for a range of transmissivity between 2 x 10^{-9} m²/s and 1 x 10^{-7} m²/s for a CCL with a typical specified hydraulic conductivity of 1 x 10^{-9} m/s. Again there is excellent agreement between the two methods of calculation, with a maximum discrepancy of less than 5% for “poor contact” (θ = 1 x 10^{-7} m²/s).

The effect of the hydraulic conductivity of the CCL over the typical range 1 x 10^{-10} to 5 x 10^{-9} m/s for good contact (θ = 1.6 x 10^{-8} m²/s) is given in Figure 4 with excellent agreement. The maximum discrepancy was 3% at k = 1 x 10^{-10} m/s.

Figure 3 Comparison of leakage through 1 cm radius hole for 2.5 holes/ha and 12 holes/ha calculated using Eq. 1 and FEM for different GM-CCL interface transmissivities. CCL hydraulic conductivity of 1 x 10^{-9} m/s.

Figure 4 Comparison of leakage through 1 cm radius hole for 2.5 holes/ha and 12 holes/ha calculated using Eq. 1 and FEM for different CCL hydraulic conductivities. Interface transmissivity of 1.6 x 10^{-8} m²/s

A similar observation can be made with respect to Figure 5 for case of “poor contact” (θ = 1 x 10^{-7} m²/s) with a maximum discrepancy being 2%. Given the uncertainty of the hydraulic conductivity (k), the difference between the analytical and finite element solution is of no practical significance.
A comparison of Figures 1 and 2 with Figures 3, 4 and 5 suggests that for the cases considered the leakage through a composite liner with GCL is likely to be 1 to 2 orders of magnitude less than for a composite liner with CCL. This is consistent with observations from double lined landfills where the leakage through the primary liner has been monitored. However, as noted by Rowe (2005) the leakage predicted assuming holes in a GM in direct contact with the clay liner considerably underestimate observed leakage unless an unrealistic number of holes are assumed. Rowe (2005) suggested that this is because of the presence of holes in wrinkles.

5.2 Leakage through a Wrinkle on flat slope (Case 2)

Leakage through a holed wrinkle, per meter length of the wrinkle was obtained using the analytical equation (Eq. 2, the Rowe equation) and SEEP/W for parallel 100m long wrinkles at a spacing 2B of 5, 10, 20, 40 and 100 meters are given in Figure 6 for the typical range of transmissivity with a GM over a GCL. Again there is a good agreement between the analytical solution and the finite element results with a discrepancy typically less than 2% and a maximum discrepancy of 5% for an interface transmissivity of 2x10^{-12} m^2/s.

Figure 7 shows a similar comparison for a range of hydraulic conductivity with a maximum discrepancy of 2%.

A comparison of Figures 6 and 7 also shows that for holed wrinkles, uncertainty regarding interface transmissivity has a much great effect on leakage than uncertainty regarding the hydraulic conductivity over ranges examined. This is consistent with the findings for holes in GMs in direct contact with the clay liner.
a discrepancy of less than 3% for “good contact” (θ = 1.6 \times 10^{-8} \text{ m}^2/\text{s}) and wrinkles spacing less than 20 m. The discrepancy was less than 2% for all wrinkle spacing (2B) larger than 20 m.

The effect of CCL hydraulic conductivity on leakage is shown for “poor contact” (θ = 1 \times 10^{-7} \text{ m}^2/\text{s}) in Figure 9. Again there is good agreement between predictions from Eq. 2 and the FEM with a maximum discrepancy of 10% for k = 1 \times 10^{-10} \text{ m/s} and wrinkle spacing’s less than 10 m. Similar findings were obtained from Figure 10 for case of “good contact” (θ = 1.6 \times 10^{-8} \text{ m}^2/\text{s}).

Comparing Figure 8 with either Figure 9 and/or Figure 10, it is seen that uncertainty regarding interface transmissivity has a much great effect on leakage than uncertainty regarding hydraulic conductivity for composite liners involving a CCL. From Figures 9, it can also be seen that the relative effect of wrinkles spacing is much greater in case of “good contact” (θ = 1.6 \times 10^{-8} \text{ m}^2/\text{s}) than for “poor contact” (θ = 1 \times 10^{-7} \text{ m}^2/\text{s}) – this is because of the much greater wetted distance from the wrinkle when the interface transmissivity is high and hence the greater interaction between adjacent wrinkles for poor contact.

### 6 CONCLUSION

The finite element method was used to calculate leakage through composite liners for the cases of (a) circular hole in a flat GM in contact with the underlying clay liner, (b) holed wrinkles in a GM. The results were compared with predictions based on the analytical solutions presented by Rowe (1998).

For the range of conditions examined (composite liners including GCLs with transmissivity in the range of 2 \times 10^{-12} \text{ m}^2/\text{s} to 2 \times 10^{-10} \text{ m}^2/\text{s}; and hydraulic conductivity of 2 \times 10^{-12} to 2 \times 10^{-10} \text{ m/s} and CCLs with transmissivity in the range of 2 \times 10^{-9} to 1 \times 10^{-7} \text{ m}^2/\text{s}; and hydraulic conductivity of 1 \times 10^{-10} to 5 \times 10^{-9} \text{ m/s}), it can be concluded that:

- For leakage through single holes, a good agreement was achieved between the analytical solution and the finite element method results. The analytical solution (Rowe, 1998) for leakage through single holes in GM, as implemented in POLLUTE V7®, gave accurate calculations of leakage (to better than 10% and usually better than 3%) for wide range of parameters and condition likely to be encountered in liner design.
- For leakage through holed wrinkles, the analytical solution (Rowe, 1998) for flow through wrinkles showed excellent agreement (to better than 11% and usually better than 2%) with the finite element method over the practical range of transmissivity and hydraulic conductivity.
- Uncertainty regarding interface transmissivity has a much great effect on leakage than uncertainty regarding hydraulic conductivity over the practical range.
For composite liners with CCLs, the relative effect of wrinkles spacing on the estimated leakage is greater in case of “good contact” than for “poor contact”.

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NOTATION
\[ A = \text{area of circular defect in geomembrane (m}^2) \]
\[ 2b = \text{width of wrinkle (m)} \]
\[ 2B = \text{distance between parallel wrinkles} \]
\[ C = \text{defined by equation 2d (-)} \]
\[ H_L = \text{thickness of the liner (m)} \]
\[ H_f = \text{thickness of the foundation layer (m)} \]
\[ H_s = \text{distance between GM and aquifer (m)} \]
\[ h_a = \text{hydraulic head in the aquifer (m).} \]
\[ h_w = \text{hydraulic head on top of geomembrane (m)} \]
\[ i_s = 1+(h_w-h_a)/(H_f+H_L) = \text{mean gradient through the clay liner and underlying foundation layer if there were no geomembrane} (-) \]
\[ k_s = \text{harmonic mean hydraulic conductivity (m s}^{-1} \]
\[ k_f = \text{hydraulic conductivity of the foundation (m s}^{-1} \]
\[ k_l = \text{the hydraulic conductivity of the liner (m s}^{-1} \]
\[ L = \text{length of an interconnected wrinkle (m)} \]
\[ Q = \text{flow through defect in geomembrane (m}^3\text{s}^{-1} \text{ for each hole or holed wrinkle; lphd for a group of holes or wrinkles)} \]
\[ \rho = \text{the interface transmissivity (m}^2\text{s}) \]
\[ r = \text{the radius of the single hole (m)} \]
\[ r_o = \text{radius of the hole in the geomembrane (m)} \]
\[ r_w = \text{the wetted radius around the hole (this can be calculated as described by Rowe (1998) or Rowe et al 2005 (m)} \]
\[ \alpha = \text{defined by equation 2b (-)} \]
\[ \theta/f \theta_0^{20} = \text{the interface transmissivity between the geomembrane and the clay liner (m}^2\text{s}^{-1} \]